DYNAMIC STABILITY OF MULTI-SPAN PIPES CONVEYING FLUID BY FINITE ELEMENT METHOD

by

RAVI KUMAR

DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

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RAVI KUMAR

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LIST OF SYMBOLS

A	Area of passage through which fluid passes.
$\lfloor B \rfloor$	Interpolation matrix.
C	Coefficient of viscous damping.
EI	Flexural rigidity of pipe.
E^*	Coefficient of internal dissipation.
f	Non-dimensional coefficient of viscous damping, $= \dfrac{CL^2}{[EI\,(M_f + m_p)]^{1/2}}$
h	Length of finite element.
K_d	Stiffness of displacement spring.
K_t	Stiffness of torsional spring.
L	Length of the pipe.
M_f	Mass of fluid/unit length.
m_p	Mass of pipe/unit length.
$\lfloor N \rfloor$	Interpolation matrix.
$\lfloor N' \rfloor$	First derivative of N with respect to ξ.
$\lfloor N'' \rfloor$	Second derivate of N with respect to ξ.
P	Fluid pressure above atmosphere.
p	Non-dimensional fluid pressure, $= \dfrac{PAL^2}{EI}$
T	Longitudinal tension.
u	Non-dimension velocity of fluid, $= (\frac{M}{EI})^{1/2}$ UL
V	Velocity of fluid.

W	Displacement in transverse direction.
w	Non-dimensional displacement, $= W/L$.
$\{w\}^n$	Nodal displacement matrix of pipe.
$\{\dot{w}\}^n$	Nodal velocity matrix of pipe.
$\{\ddot{w}\}^n$	Nodal acceleration matrix of pipe.
$\{w\}^{ne}$	Nodal displacement matrix of finite element.
$\{\dot{w}\}^{ne}$	Nodal velocity matrix of finite element.
$\{\ddot{w}\}^{ne}$	Nodal acceleration matrix of finite element.
X	Coordinate along length of the pipe.
x	Non-dimensional X coordinate, $= X/L$
α	Non-dimensional coefficient of internal dissipation, $= \left[\frac{I}{E(M_f+m_p)}\right]^{1/2} \frac{E^*}{L^2}$.
α_d	Non-dimensional stiffness of displacement spring, $= \frac{K_d L^3}{EI}$
α_t	Non-dimensional stiffness of torsional spring, $\frac{K_t L}{EI}$.
β	Non-dimensional mass ratio parameter, $(M_f/(M_f+m_p))$
γ	Non-dimensional gravity, $= \frac{M_f+m_p}{EI} L^3 g$
δ	Excitation parameter.
Γ	Non-dimensional longitudinal tension, $= \frac{TL^2}{EI}$.
ξ	Local coordinate of finite element.
ξ_1	Length coordinate, $= 1 - \xi/h$.
ξ_2	Length coordinate, $= \xi/h$

ω	Frequency.
Ω	Non-dimensional frequency, $= \left(\frac{M_f + m_p}{EI}\right)^{1/2} L^2$.
ν	Poisson's ratio.
$\mu = 0$	If no axial constraint at end.
$\mu = 1$	If there is axial constraint at end
τ	Non-dimensional time, $= \left(\frac{EI}{M_f + m_p}\right)^{1/2} \frac{t}{L^2}$.

SYNOPSIS

This thesis deals with dynamic stability of multi-span pipes conveying fluid where the flow velocity is either constant or a small harmonic component is superimposed on it. The very general equation of motion is used to study the stability of the pipes. The element matrices are obtained by the finite element (Galerkin) method. For steady flow stiffness, damping and mass matrices are obtained by finite element methods and a standard dynamical matrix is obtained. The critical velocities are obtained by solving eigen value problem. For harmonically perturbed flow the bounds for the principal primary regions of instability are determined by Bolotin's method. It is shown that critical velocities and instability regions can be controlled by changing the position of the supports.

Two general computer programs have been written to solve these problems. These programs are very flexible in nature and solution for various types of multi-span pipes can very easily be obtained by changing the input data.

CHAPTER - I

INTRODUCTION

1.1 INTRODUCTION

Pipes conveying fluids are encountered in various fields of engineering. To name some of them ; oil pipe lines, propellent lines, pump discharge lines, heat exchanger tubes and coolant channels of nuclear reactors. An imperfect design of pipe lines may result in a leakage and unsatisfactory performance of the whole system. As the flow velocity is increased a certain velocity is reached when the pipe either buckles or flutters. With harmonic perturbations these systems are vulnerable to parametric instability also.

In the recent years many pipe lines have been laid and this has accelerated the research work in this field. Lot of research has been done in the last thirty years. However most of literature is devoted to either single span pipe or periodic pipes. In actual practice however we come across nonperiodic multi-span pipes.

The aim of the present work is to study the stability of multi-span pipes conveying fluid at a

constant velocity or when a small harmonic component is superimposed on it, by finite element method. All types of boundary conditions can easily be accommodated by this method. The objective of present thesis is explained in detail in the last section of this chapter.

2.2 REVIEW OF PREVIOUS WORK

A detailed review of the literature with extensive references is given in Paidoussis and Issid [21] and Chen [6]. A brief review is given below.

Interest in the subject of dynamics of pipes conveying fluid was developed in 1950 in connection with study of vibration of Trans Arabian pipe by Ashley and Haviland [1]. They studied the problem of vibration of simply supported pipe. The same problem was studied by Housner [12] independently by using a different approach. He found that at low fluid velocities the effect upon vibrations of the pipe was negligible but at certain high velocities pipe buckled like a column subjected to axial loading.

Long [15] considered the case of cantilever pipe conveying fluid. He adopted an iterative procedure using a power series for mode shape, which was applicable to relatively small flow velocities and confirmed experimentally that forced motion of cantilever pipes was damped

by internal flow in the range of flow velocities considered.

Benjamin [2] dealing with the general dynamical problem of cantilevered system of articulated pipes conveying fluid was first to anticipate the phenomenon of instable oscillations (flutter). He produced complete theory supported by experiments for articulated pipe systems. He showed that when the system was vertical both oscillation and buckling instabilities are possible. However in general when motion is confined to horizontal plane, that is gravity is insignificant, buckling can not occur. Later Gregory and Paidoussis [10] confirmed both theoretically and experimentally that at sufficiently high velocity cantilever pipes are subjected to oscillatory instabilities. Next Paidoussis [19] included the effect of gravity and showed that in case of vertical cantilevers buckling instability is not possible. This contradiction was cleared by Paidoussis and Deskins [2o].

Naguleswaran and William [17] studied both theoretically and experimentally the effect of fluid pressure on the dynamics of the pipe. They showed that pipes with both ends supported may buckle at small flow velocities by the action of high internal pressure.

Chen [5] studied the stability of pipes conveying fluid with upstream end fixed and downstream end supported

by displacement springs, so that boundary conditions are intermediate between clamped free and clamped-pinned. He showed that both buckling and oscillary instabilities are possible depending upon the spring constant.

Paidoussis and Issid [21] derived the general equation of motion. They included the effect of axial movement of the pipe. They studied the dynamic stability of the pipe where the flow velocity is either constant or a small harmonic component is superimposed on it. They showed that with both ends fixed pipe is subjected not only to buckling instability but flutter instability is also possible at high velocities.

Singh and Mallik [22] studied the wave propogation and vibration response of a periodically supported pipe by propogation constant approach. A succeeding paper by same authors [23] was devoted to parametric instabilities.

Orris and Petyt [18] applied the finite element methods to propogation constant in periodic structures. Deb [7] applied finite element (Galerkin) method and obtained flutter and buckling instabilities of various types of single span pipes.

Kulkarni [13] studied instabilities of the periodic pipes due to parametric excitation considering the effect of axial movement of the pipe. He used the propogation constant technique.

1.3 OBJECTIVE AND SCOPE OF PRESENT WORK

The aim of the present work is to develop finite element method for studying the dynamic stability of multi-span pipes conveying fluid.

In second chapter element matrices are obtained by the finite element (Galerkin method) for pipes conveying fluid when the flow velocity is either constant or a small harmonic component superimposed on it. For steady flow stiffness, damping and mass matrices are obtained by the finite element method and a standard dynamical matrix obtained. Critical velocities are obtained by solving the eigen-value problem. For harmonically perturbed flow, bounds of principal primary instability regions are obtained by using Bolotin's method.

In third chapter results for various types of pipe configurations are given and discussed.

Conclusions are reported in the last chapter. The values or various matrices and listing of computer program are given in Appendixes.

CHAPTER - II

GENERAL FORMULATION

This chapter deals with the finite element analysis of stability of pipes conveying fluid at constant velocity V and with a small harmonic component superimposed on it.

2.1 EQUATION OF MOTION

Consider a pipe conveying fluid at velocity V, Fig. 1. Its equation of motion is, Paidoussis and Issid [21]

$$E^{*}I \frac{\partial^5 W}{\partial X^4 \partial t} + EI \frac{\partial^4 W}{\partial X^4} + \left\{ M_f V^2 - T + PA (1 - 2\nu\mu) - \left[(M_f + m_p) g - M_f \frac{\partial V}{\partial t} \right] (L - X) \right\} \frac{\partial^2 W}{\partial X^2}$$

$$+ 2 M_f V \frac{\partial^2 W}{\partial X \partial t} + (M_f + m_p)\, g \frac{\partial W}{\partial X} + c \frac{\partial W}{\partial t}$$

$$+ (M_f + m_p) \frac{\partial^2 W}{\partial t^2} = 0 \qquad (2.1)$$

where

E^{*}	Coefficient of internal dissipation
EI	Flexural rigidity of pipe
M_f	Mass of fluid per unit length
V	Velocity of the fluid

T	Longitudinal tension
P	Fluid pressure above atmosphere
ν	Poisson's ratio
m_p	Mass of pipe per unit length
L	Total length of the pipe
c	Coefficient of viscous damping
$\mu=0$	For no axial constraint at the support
$\mu=1$	For axial constraint at the support

.... (2.2)

The equation (2.1) can be expressed in dimensionless form by defining the following quantities:

$$x = \frac{X}{L}, \quad w = \frac{W}{L}, \quad \tau = \left(\frac{EI}{M_f + m_p}\right)^{1/2} \frac{1}{L^2}$$

$$\alpha = \left[\frac{I}{E(M_f+m_p)}\right]^{1/2} \frac{E^*}{L^2}, \quad u = \left(\frac{M_f}{EI}\right)^{1/2} VL, \quad \beta = \frac{M_f}{M_f + m_p}$$

$$\gamma = \frac{M_f + m_p}{EI} L^3 g, \quad \Gamma = \frac{TL^2}{EI}, \quad f = \frac{cL^2}{[EI(M_f+m_p)]^{1/2}}$$

$$p = \frac{PAL^2}{EI} \tag{2.3}$$

Substituting equation (2.3) into equation (2.1), the equation of motion becomes

$$\alpha \frac{\partial^5 w}{\partial x^4 \partial \tau} + \frac{\partial^4 w}{\partial x^4} + \left\{ u^2 - \Gamma + p(1 - 2\nu\mu) + \left(\beta^{1/2} \frac{\partial u}{\partial \tau} - \gamma\right)x \right.$$
$$\left. x(1-x) \right\} \frac{\partial^2 w}{\partial x^2} + 2\beta^{1/2} u \frac{\partial^2 w}{\partial x \partial \tau} + \gamma \frac{\partial w}{\partial x} + f \frac{\partial w}{\partial \tau} + \frac{\partial^2 w}{\partial \tau^2} = 0$$

... (2.4)

2.2 PIPES WITH STEADY FLOW

For constant velocity, equation (2.4) becomes

$$\alpha \frac{\partial^5 w}{\partial x^4 \partial \tau} + \frac{\partial^4 w}{\partial x^4} + \{u^2 - \Gamma + p\,(1-2\nu\mu) - \gamma\,(1-x)\} \frac{\partial^2 w}{\partial x^2}$$

$$+ 2\beta^{1/2} u \frac{\partial^2 w}{\partial x \partial \tau} + \gamma \frac{\partial w}{\partial x} + f \frac{\partial w}{\partial \tau} + \frac{\partial^2 w}{\partial \tau^2} = 0 \qquad (2.5)$$

2.2.1 Finite Element Analysis:

For finite element solution (Galerkin Method) of this one dimensional partial differential equation, the pipe is divided into number of finite elements, Figure 2. Typical finite element is shown in Figure 3. Its solution is assumed to be of the form of

$$w^{(e)}(\xi, \tau) = \lfloor N_1\ N_2 \ldots\ldots N_r \rfloor \begin{Bmatrix} w_1 \\ w_2 \\ \vdots \\ w_r \end{Bmatrix}$$

$$\text{or} \quad w^{(e)}(\xi, \tau) = \lfloor N_i(\xi) \rfloor \{w_i(\tau)\}^{(ne)} \qquad (2.6)$$

where

- N_i Interpolating functions of the element
- w_i Nodal parameters of the element
- r Degrees of freedom of the element (to be decided later)
- ξ Local co-ordinate of the element.

Substituting equation (2.6) into equation (2.5), one gets the residue for the element,

$$R^{(e)} = \alpha \frac{\partial^5 w^{(e)}}{\partial \xi^4 \ \partial \tau} + \frac{\partial^4 w^{(e)}}{\partial \xi^4} + \{u^2 - \Gamma + p\ (1-2\nu\mu) - \gamma\} \frac{\partial^2 w^{(e)}}{\partial \xi^2}$$
$$+ \gamma\ (x_j + \xi) \frac{\partial^2 w^{(e)}}{\partial \xi^2} + \gamma \frac{\partial w^{(e)}}{\partial \xi} + 2\beta^{1/2}\ u \frac{\partial^2 w^{(e)}}{\partial \xi\ \partial \tau}$$
$$+ f \frac{\partial w^{(e)}}{\partial \tau} + \frac{\partial^2 w^{(e)}}{\partial \tau^2} \tag{2.7}$$

Minimizing this residue by the Galerkin method,

$$\int_0^h N_i\ R^{(e)}\ d\xi = 0$$

or
$$\int_0^h N_i\ \Big(\alpha \frac{\partial^5 w(e)}{\partial \xi^4\ \partial \tau} + \frac{\partial^4 w^{(e)}}{\partial \xi^4} + \{u^2 - \Gamma + p\ (1-2\nu\mu)$$
$$- \gamma\} \frac{\partial^2 w(e)}{\partial \xi^2} + \gamma\ (x_j + \xi) \frac{\partial^2 w^{(e)}}{\partial \xi^2} + \gamma \frac{\partial w^{(e)}}{\partial \xi}$$
$$+ 2\beta^{1/2}\ u \frac{\partial^2 w^{(e)}}{\partial \xi \partial \tau} + f \frac{\partial w^{(e)}}{\partial \tau} + \frac{\partial^2 w^{(e)}}{\partial \tau^2} \Big)\ d\xi = 0 \tag{2.8}$$

In order to reduce the requirements of interpolating polynomials, the first four terms of equation (2.8) are integrated by parts, and one gets

$$N_i\ \alpha \frac{\partial^3}{\partial \xi^3} \left(\frac{\partial w^{(e)}}{\partial \tau}\right) \Big|_0^h - \frac{\partial N_i}{\partial \xi}\ \alpha \frac{\partial^2}{\partial \xi^2} \left(\frac{\partial w^{(e)}}{\partial \tau}\right) \Big|_0^h + \int_0^h \frac{\partial^2 N_i}{\partial \xi^2}\ \alpha \frac{\partial^3 w^{(e)}}{\partial^2 \xi\ \partial \tau} d\xi$$
$$+ N_i \frac{\partial^3 e^{(e)}}{\partial \xi^3} \Big|_0^h - \frac{\partial N_i}{\partial \xi} \frac{\partial^2 w^{(e)}}{\partial \xi^2} \Big|_0^h + \int_0^h \frac{\partial^2 N_i}{\partial \xi^2} \frac{\partial^2 w(e)}{\partial \xi}\ d\xi$$
$$+ N_i \{u^2 - \Gamma + p\ (1-2\nu\mu) - \gamma\} \frac{\partial w^{(e)}}{\partial \xi} \Big|_0^h$$
$$- \{u^2 - \Gamma + p\ (1-2\nu\mu) - \gamma\} \int_0^h \frac{\partial N_i}{\partial \xi} \frac{\partial w^{(e)}}{\partial \xi} d\xi$$
$$+ \gamma\ x_j\ N_i \frac{\partial w^{(e)}}{\partial \xi} \Big|_0^h - \gamma\ x_j \int_0^h \frac{\partial N_i}{\partial \xi} \frac{\partial w^{(e)}}{\partial \xi} d\xi + \gamma \xi\ N_i \frac{\partial w^{(e)}}{\partial \xi} \Big|_0^h$$

$$- \gamma \int_0^h \frac{\partial N_i}{\partial \xi} \xi \frac{\partial w^{(e)}}{\partial \xi} d\xi + \int_0^h N_i \; 2\beta^{1/2} \; u \frac{\partial^2 w^{(e)}}{\partial \xi \, \partial \tau} d\xi$$

$$+ f \int_0^h N_i \frac{\partial w^{(e)}}{\partial \tau} d\xi + \int_0^h N_i \frac{\partial^2 w^{(e)}}{\partial \tau^2} d\xi = 0 \qquad (2.9)$$

Highest derivative of $w^{(e)}$ with respect to ξ in any integral in equation (2.9) is second. Thus interpolating function N_i should have compatibility of $w^{(e)}$ and $\frac{\partial w^{(e)}}{\partial \xi}$. Highest derivative in equation (2.9) is third. Thus interpolating function N_i should have completeness of $w^{(e)}$, $\frac{\partial w^{(e)}}{\partial \xi}$, $\frac{\partial^2 w}{\partial \xi^2}$ and $\frac{\partial^3 w^{(e)}}{\partial \xi^3}$. The function satisfying these requirements of compatibility and completeness is

$$w^{(e)} = a + bx + cx^2 + dx^3$$

or $$w^{(e)} = \lfloor \xi_1^2 \, (3-2\xi_1), h\xi_1^2\xi_2 \, , \; \xi_2^2 \, (3-2 \;_2), -h\xi_1 \xi_2^2 \rfloor \begin{Bmatrix} w_j \\ w_j' \\ w_k \\ w_k' \end{Bmatrix} \quad \ldots \quad (2.10)$$

or $$w^{(e)} = \lfloor N \, (\xi) \rfloor \{ w \, (\tau) \}^{ne}$$

where $\xi_1 = 1 - \xi/h$, $\xi_2 = \xi/h$

that is the typical element has four degrees of freedom.

Using equation (2.10), the equation (2.9) can be written in the following matrix form

$$[m]^{(e)}\{\ddot{w}\}^{(ne)} + [c]^{(e)}\{\dot{w}\}^{(ne)} + [k]^{(e)}\{w\}^{(ne)}$$

$$= -\left\{N\,\alpha\,\frac{\partial^3}{\partial\xi^3}\left(\frac{\partial w^{(e)}}{\partial\tau}\right)\Big|_0^h\right\} + \left\{\frac{\partial N}{\partial\xi}\,\alpha\,\frac{\partial^2}{\partial\xi^2}\left(\frac{\partial w^{(e)}}{\partial\tau}\right)\Big|_0^h\right\}$$

$$-\left\{N\,\frac{\partial^3 w^{(e)}}{\partial\xi^3}\Big|_0^h\right\} + \left\{\frac{\partial N}{\partial\xi}\,\frac{\partial^2 w^{(e)}}{\partial\xi^2}\Big|_0^h\right\}$$

$$-\left\{N(u^2-\Gamma + p\,(1-2\nu\mu) - \gamma)\,\frac{\partial w^{(e)}}{\partial\xi}\Big|_0^h\right\}$$

$$-\left\{\gamma\,N\,(x_j+\xi)\,\frac{\partial w^{(e)}}{\partial\xi}\right\}\Big|_0^h \qquad (2.11)$$

where

$$[m]^{(e)} = \int_0^h \{N\}\lfloor N \rfloor\, d\xi \qquad (2.12)$$

$$[c]^{(e)} = \alpha\int_0^h \{N''\}\lfloor N''\rfloor d\xi + 2\beta^{1/2}\,u\int_0^h\{N\}\lfloor N'\rfloor d\xi$$

$$+ f\int_0^h \{N\}\lfloor N\rfloor\, d\xi \qquad (2.13)$$

$$[k]^{(e)} = \int_0^h\{N''\}\lfloor N''\rfloor d\xi$$

$$-(u^2-\Gamma + p(1-2\nu\mu) - \gamma)\int_0^h\{N'\}\lfloor N'\rfloor d\xi$$

$$-\gamma x_j\int_0^h\{N'\}\lfloor N'\rfloor d\xi - \gamma\int_0^h\{N'\}\lfloor N'\rfloor\, d\xi \qquad (2.14)$$

Values of matrices $[m]^{(e)}$, $[c]^{(e)}$ and $[k]^{(e)}$ are given in Appendix I. Using the values of N_i from equation (2.10) in the right side of equation (2.11), one gets

$$[m]^{(e)}\left\{\ddot{w}\right\}^{(ne)} + [c]^{(e)}\left\{\dot{w}\right\}^{(ne)} + [k]^{(e)}\left\{w\right\}^{(ne)}$$

$$= \begin{Bmatrix} \alpha \left.\dfrac{\partial^4 w^{(e)}}{\partial \xi^3\, \partial \tau}\right|_j \\ 0 \\ -\alpha \left.\dfrac{\partial^4 w^{(e)}}{\partial \xi^3\, \partial \tau}\right|_k \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\alpha \left.\dfrac{\partial^3 w^{(e)}}{\partial \xi^2\, \partial \tau}\right|_j \\ 0 \\ \alpha \left.\dfrac{\partial^3 w^{(e)}}{\partial \xi^2\, \partial \tau}\right|_k \end{Bmatrix} + \begin{Bmatrix} \left.\dfrac{\partial^3 w}{\partial \xi^3}\right|_j \\ 0 \\ -\left.\dfrac{\partial^3 w^{(e)}}{\partial \xi^3}\right|_k \\ 0 \end{Bmatrix}$$

$$+ \begin{Bmatrix} 0 \\ -\left.\dfrac{\partial^2 w^{(e)}}{\partial \xi}\right|_j \\ 0 \\ \left.\dfrac{\partial^2 w^{(e)}}{\partial \xi^2}\right|_k \end{Bmatrix} + \begin{Bmatrix} (u^2 - \Gamma + p(1-2\nu\mu) - \gamma) \left.\dfrac{\partial w^{(e)}}{\partial \xi}\right|_j \\ 0 \\ -(u^2 - \Gamma + p(1-2\nu\mu) - \gamma) \left.\dfrac{\partial w}{\partial \xi}\right|_k \\ 0 \end{Bmatrix} + \begin{Bmatrix} \gamma x \left.\dfrac{\partial w(e)}{\partial \xi}\right|_j \\ 0 \\ -\gamma x \left.\dfrac{\partial w^{(e)}}{\partial \xi}\right|_k \\ 0 \end{Bmatrix} \quad \ldots \qquad (2.15)$$

The equation (2.15) is for one finite element. These are assembled for the whole domain,

$$[M]\left\{\ddot{w}\right\}^{(n)} + [C]\left\{\dot{w}\right\}^{(n)} + [K]\left\{w\right\}^{(n)} =$$

$$
\begin{Bmatrix} \alpha \left.\dfrac{\partial^4 w^{(e)}}{\partial \xi^3 \, \partial \tau}\right|_1 \\ 0 \\ \vdots \\ -\alpha \left.\dfrac{\partial^4 w^{(e)}}{\partial \xi^3 \, \partial \tau}\right|_{n+1} \\ 0 \end{Bmatrix}
+ \begin{Bmatrix} 0 \\ -\alpha \left.\dfrac{\partial^3 w^{(e)}}{\partial \xi^2 \, \partial \tau}\right|_1 \\ \vdots \\ 0 \\ \alpha \left.\dfrac{\partial^3 w^{(e)}}{\partial \xi^2 \, \partial \tau}\right|_{n+1} \end{Bmatrix}
+ \begin{Bmatrix} \left.\dfrac{\partial^3 w^{(e)}}{\partial \xi^3}\right|_1 \\ 0 \\ \vdots \\ -\left.\dfrac{\partial^3 w}{\partial \xi^3}\right|_{n+1} \\ 0 \end{Bmatrix}
$$

$$
+ \begin{Bmatrix} 0 \\ -\left.\dfrac{\partial^2 w^{(e)}}{\partial \xi^2}\right|_1 \\ \vdots \\ 0 \\ \left.\dfrac{\partial^2 w}{\partial \xi^2}\right|_{n+1} \end{Bmatrix}
+ \begin{Bmatrix} (u^2 - \Gamma + p(1-2\nu\mu) - \gamma) \left.\dfrac{\partial w^{(e)}}{\partial \xi}\right|_1 \\ 0 \\ \vdots \\ -(u^2 - \Gamma + p(1-2\nu\mu) - \gamma) \left.\dfrac{\partial w^{(e)}}{\partial \xi}\right|_{n+1} \\ 0 \end{Bmatrix} .
$$

$$
+ \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ -\gamma \; nh \left.\dfrac{\partial w^{(e)}}{\partial \xi}\right|_{n+1} \\ 0 \end{Bmatrix} \tag{2.16}
$$

After applying boundary conditions, the equation (2.16) reduces to

$$[M]_{BC}\{\ddot{w}\}^{(n)} + [C]_{BC}\{\dot{w}\}^{(n)} + [K]_{BC}\{w\}^{(n)} = 0 \quad \ldots \quad (2.17)$$

For boundary conditions, see Appendix 2.

2.2.2 Method of Solution:

Equations (2.17) are set of homogenous differential equations. The solution of this homogenous set of differential equations is obtained as detailed in Meirovitch (14]. But it may be noted that the matrices $[M]_{BC}$, $[C]_{BC}$ and $[K]_{BC}$ need not be symmetric as stated in Meirovitch [14], see Frazer, Duncan and Collar [9]. Using generalised velocity $\dot{w}$ as auxiliary variables, n second order ordinary differential equations (2.17) are converted to a set of 2n first order ordinary differential equations,

$$[\bar{M}]\{\dot{\eta}(\tau)\} + [\bar{K}]\{\eta(\tau)\} = \{0\} \quad (2.18)$$

where

$$\{\eta(\tau)\} = \begin{Bmatrix} \{w(\tau)\} \\ \{\dot{w}(\tau)\} \end{Bmatrix} \quad (2.19)$$

and

$$[\bar{M}] = \begin{bmatrix} [0] & [M]_{BC} \\ [M]_{BC} & [C]_{BC} \end{bmatrix} \text{and} \quad [\bar{K}] = \begin{bmatrix} -[M]_{BC} & [0] \\ [0] & [K]_{BC} \end{bmatrix} \quad \ldots \quad (2.20)$$

Now we have the differential equations which in the matrix form are

$$[\bar{M}] \left\{ \dot{\eta}(\tau) \right\} + [\bar{K}] \left\{ \eta(\tau) \right\} = \left\{ 0 \right\} \tag{2.21}$$

$$\text{set} \left\{ \eta(\tau) \right\} = e^{\Psi} \left\{ \emptyset \right\} \tag{2.22}$$

where $\emptyset$ consists of a vector consisting of 2n constant terms. Substitution of equation (2.22) in equation (2.21) leads to the eigen-value problem

$$\Psi [\bar{M}] \left\{ \emptyset \right\} + [\bar{K}] \left\{ \emptyset \right\} = 0 \tag{2.23}$$

which can be written as

$$[D] \left\{ \emptyset \right\} = \frac{1}{\Psi} \left\{ \emptyset \right\} \tag{2.24}$$

where

$$(D] = - [\bar{K}]^{-1} [\bar{M}] = \begin{bmatrix} [0] & [I] \\ -[K]^{-1}[M] & -[K]^{-1}[C] \end{bmatrix} \tag{2.25}$$

The eigen value of equation (2.23) will in general be complex. The real part of eigen value determines the stability of pipe and imaginary part gives the frequency of pipe. If the real part is positive and imaginary part is nonzero then there will be flutter. If real part is positive and imaginary part is zero the pipe will buckle. The velocity u at which pipe fails either due to buckling or flutter is known as the critical velocity, u_c.

A library subroutine EIGRF was used to calculate the complex eigen values.

2.3 PIPES WITH HARMONICALLY PERTURBED FLOW

Equation (2.4) is general governing differential equation of the pipe conveying fluid.

Now a case is considered where the flow velocity is perturbed harmonically, that is

$$u = u_o \,(1 + \delta \cos \Omega \tau) \tag{2.26}$$

where u_o is mean flow velocity, δ is excitation parameter and Ω is non-dimensional frequency given by $\frac{\omega t}{\tau}$.

Substituting equation (2.26) in equation (2.4) one gets

$$\alpha \frac{\partial^5 w}{\partial x^4 \partial \tau} + \frac{\partial^4 w}{\partial x^4} + [\, u_o^2 (1 + \delta \cos \Omega \tau)^2 - + p\,(1-2\nu\mu)$$

$$-\gamma\,(1 - x) - \beta^{1/2} u_o \,\delta \Omega\,(1 - x) \sin \Omega \tau] \frac{\partial^2 w}{\partial x^2}$$

$$+2\beta^{1/2} u_o \,(1 + \delta \cos \Omega \tau) \frac{\partial^2 w}{\partial x \,\partial \tau} + \gamma \frac{\partial w}{\partial x} + f \frac{\partial w}{\partial \tau}$$

$$+ \frac{\partial^2 w}{\partial \tau^2} = 0 \tag{2.27}$$

The coefficient of some derivatives are time varying, hence the pipe is parametrically excited. Because of parametric excitation, instability will occur over a range of frequencies.

In order to obtain the primary instability regions, we can write, Bolotin [4]

$$w(x,\tau) = \Sigma\,(Y_k\,(x)\,\sin\,(\frac{\Omega\,\tau}{2}) + Z_k\,(x)\,\cos\,(\frac{\Omega\,\tau}{2})$$

$$k = 1,3,5\ldots\ldots \qquad (2.28)$$

The regions of primary instabilities can be obtained quite accurately by taking K = 1 approximation, Paidoussis and Issid [21]. Hence one can write

$$w\,(x\,,\tau) = Y\,(x)\,\sin\,(\frac{\Omega\,\tau}{2}) + Z\,(x)\,\cos\,(\frac{\Omega\,\tau}{2}) \qquad (2.29)$$

Substituting equation (2.29) in equation (2.27) and equating the coefficients of sin $(\frac{\Omega\,\tau}{2})$ and cos $(\frac{\Omega\,\tau}{2})$ separately to zero, one gets

$$\frac{d^4Y}{dx^4} + \left(u_o^2\,(1 - \delta + \frac{\delta^2}{2}) - \Gamma + p\,(1 - 2\nu\mu) - \gamma\,(1-x)\right)\frac{d^2Y}{dx^2}$$

$$- \frac{1}{4}\,\Omega^2\,Y + \gamma\,\frac{dY}{dx} - \frac{\beta^{1/2}\,u_o\,\delta\,\Omega}{2}\left[(1-x)\,\frac{d^2Z}{dx^2} - (1 - \frac{2}{\delta})\,\frac{dZ}{dx}\right]$$

$$- \frac{\alpha\,\Omega}{2}\,\frac{d^4Z}{dx^4} - \frac{f\,\Omega}{2}\,Z = 0 \qquad (2.30a)$$

$$\frac{d^4Z}{dx^4} + \left(u_o^2\,(1 + \delta + \frac{\delta^2}{2}) - \Gamma + p\,(1 - 2\nu\mu) - \gamma\,(1 - x)\right)\frac{d^2Z}{dx^2}$$

$$- \frac{1}{4}\,\Omega^2\,Z + \gamma\,\frac{dZ}{dx} - \frac{\beta^{1/2}u_o\,\delta\Omega}{2}\left[(1 - x)\,\frac{d^2Y}{dx^2} - (1 + \frac{2}{\delta})\,\frac{dY}{dx}\right]$$

$$+ \frac{\alpha\,\Omega}{2}\,\frac{d^4Y}{dx^4} + \frac{f\,\Omega}{2}\,Y = 0 \qquad (2.30b)$$

Let us take

$$A_1 = u_o^2\,(1 - \delta + \frac{\delta^2}{2}) - \Gamma + p\,(1 - 2\nu\,) - \gamma \qquad (2.31)$$

$$A_2 = \frac{1}{4}\,\Omega^2 \qquad (2.32)$$

$$A_3 = \frac{\beta^{1/2} u_o \delta \Omega}{2} \tag{2.33}$$

$$A_4 = (1 - \frac{2}{\delta}) \frac{\beta^{1/2} u_o \delta \Omega}{2} \tag{2.34}$$

$$A_5 = u_o^2 (1 + \delta + \frac{\delta^2}{2}) - + p (1 - 2\nu\mu) - \gamma \tag{2.35}$$

$$A_6 = (1 + \frac{2}{\delta}) \frac{\beta^{1/2} U_o \delta \Omega}{2} \tag{2.36}$$

Using equations (2.31) to (2.36), equations (2.30) become

$$\frac{d^4Y}{dx^4} + A_1 \frac{d^2Y}{dx^2} - A_2 Y + \gamma \frac{d}{dx} (x \frac{dY}{dx}) - A_3 (1 - x) \frac{d^2Z}{dx^2}$$

$$+ A_4 \frac{dZ}{dx} - \frac{\alpha \Omega}{2} \frac{d^4Z}{dx^4} - \frac{f \Omega}{2} Z = 0 \tag{2.37a}$$

$$\frac{d^4Z}{dx^4} + A_5 \frac{d^2Z}{dx^2} - A_2 Z + \gamma \frac{d}{dx} (x \frac{dZ}{dx}) - A_3 (1 - x) \frac{d^2Y}{dx^2}$$

$$+ A_6 \frac{dY}{dx} + \frac{\alpha \Omega}{2} \frac{d^4Y}{dx^4} + \frac{f \Omega}{2} Y = 0 \tag{2.37b}$$

2.3.1 Finite Element Analysis:

Equations (2.37) are coupled differential equations. For finite element solution of these two coupled differential equations, the pipe is divided into number of finite elements, Figure 2. Solution over the finite element is assumed of the form of

$$Y^{(e)} (\xi) = \lfloor N_i (\xi) \rfloor \{ Y_i \}^{(ne)} \tag{2.38a}$$

and

$$Z^{(e)}(\xi) = \lfloor B_i(\xi) \rfloor \{ Z_i \}^{(ne)} \qquad (2.38b)$$

where,

N_i , B_i Interpolating functions

Y_i , Z_i Nodal parameters

ξ Local coordinate

Substituting equation (2.38) in (2.37) one gets the residues $R_a^{(e)}$ and $R_b^{(e)}$ for equation (2.37a) and (2.37b) respectively.

$$R_a^{(e)} = \frac{d^4Y^{(e)}}{d\xi^4} + A_1 \frac{d^2Y^{(e)}}{d\xi^2} - A_2 Y^{(e)} + \gamma \frac{d}{d\xi}\left((x_j+\xi)\frac{dY^{(e)}}{d\xi}\right)$$

$$- A_3 (1 - x_j - \xi) \frac{d^2Z^{(e)}}{d\xi^2} + A_4 \frac{dZ^{(e)}}{d\xi} - \frac{\alpha\ \Omega}{2} \frac{d^2Z}{d\xi^4} - \frac{f\ \Omega}{2} Z^{(e)}$$

$$\ldots\ldots \qquad (2.39a)$$

$$R_b^{(e)} = \frac{d^4Z^{(e)}}{d\xi^4} + A_5 \frac{d^2Z^{(e)}}{d\xi^2} - A_2 Z^{(e)} + \gamma \frac{d}{d\xi}\left((x_j+\xi)\frac{dZ^{(e)}}{d\xi}\right)$$

$$- A_3 (1 - x_j - \xi) \frac{d^2Y^{(e)}}{d\xi^2} + A_6 \frac{dY^{(e)}}{d\xi} + \frac{\alpha\ \Omega}{2} \frac{d^4Y^{(e)}}{d\xi^4} + \frac{f\ \Omega}{2} Y^{(e)}$$

$$\ldots\ldots \qquad (2.39b)$$

where x_j is the global coordinate of the j^{th} node.

Minimizing the residues by the Galerkin method

$$\int_0^h N_i R_a^{(e)}\ d\xi = 0 \qquad (2.40a)$$

$$\int_0^h B_i R_b^{(e)}\ d\xi = 0 \qquad (2.40b)$$

Treatment for (2.40a) and (2.40b) is similar. So details are given only for (2.40a) and results of both are given in equation (2.46).

Substituting the value of $R_a^{(e)}$ from equation (2.39a) in the equation (2.40a), one gets

$$\int_0^h N_i \left(\frac{d^4Y^{(e)}}{d\xi^4} + A_1 \frac{d^2Y^{(e)}}{d\xi^2} - A_2 Y^{(e)} + \gamma \frac{d}{d\xi}\left((x_j+\xi)\frac{dY^{(e)}}{d\xi}\right)\right.$$

$$\left. - A_3 (1 - x_j - \xi) \frac{d^2Z^{(e)}}{d\xi^2} + A_4 \frac{dZ^{(e)}}{d\xi} - \frac{\alpha\ \Omega}{2} \frac{d^4Z^{(e)}}{d\xi^4} - \frac{f\ \Omega}{2} Z^{(e)}\right) d\xi$$

$$= 0 \qquad \ldots\ldots \qquad (2.41)$$

In order to reduce the requirements of the enterpolating polynomials first, second, fourth, fifth and seventh terms are integrated by parts, and one gets

$$\int_0^h \frac{d^2N_i}{d\xi} \frac{d^2Y^{(e)}}{d\xi^2} d\xi - A_1 \int_0^h \frac{dN_i}{d\xi} \frac{dY^{(e)}}{d\xi} d\xi - A_2 \int_0^h N_i Y\, d\xi$$

$$- \gamma\, x_j \int_0^h \frac{dN_i}{d\xi} \frac{dY^{(e)}}{d\xi} d\xi - \gamma \int_0^h \xi \frac{dN_i}{d\xi} \frac{dY^{(e)}}{d\xi}\, d\xi$$

$$+ A_3 (1 - x_j) \int_0^h \frac{dN_i}{d\xi} \frac{dZ^{(e)}}{d\xi} d\xi - A_3 \int_0^h \frac{dN_i}{d\xi} \frac{dZ^{(e)}}{d\xi} d\xi$$

$$- A_3 \int_0^h N_i \frac{dZ^{(e)}}{d\xi} d\xi + A_4 \int_0^h N_i \frac{dZ^{(e)}}{d\xi} d\xi$$

$$- \frac{\alpha\ \Omega}{2} \int_0^h \frac{d^2N_i}{d\xi^2} \frac{d^2Z^{(e)}}{d\ \xi^2} d\xi \quad - \frac{f\ \Omega}{2} \int_0^h N_i\, Z^{(e)}\, d\xi$$

$$N_i \frac{d^3Z^{(e)}}{d\ \xi^3} \Big|_0^h - \frac{dN_i}{d\xi} \frac{d^2Y^{(e)}}{d\xi^2} \Big|_0^h + A_1\, N_i \frac{dY^{(e)}}{d\xi} \Big|_0^h$$

$$- A_3 (1 - x_j - \xi) N_i \frac{Z^{(e)}}{d\xi} \Big|_0^h - \gamma N_i (x_j + \xi) \frac{dY^{(e)}}{d\xi} \Big|_0^h$$

$$- \frac{\alpha \Omega}{2} N_i \frac{d^3 Z^{(e)}}{d\xi^3} \Big|_0^h + \frac{\alpha \Omega}{2} \frac{dN_i}{d\xi} \frac{d^2 Z^{(e)}}{d\xi^2} \Big|_0^h = 0 \qquad (2.42a)$$

Similar equation (2.42b) is obtained from equation (2.40b). Highest order of derivative in the integrals in equations (2.42) is two. Therefore $Y^{(e)}$ and $Z^{(e)}$ should have compatibility of $(Y, \frac{dY}{d\xi})$ and $(Z, \frac{dZ}{d\xi})$ respectively. Highest order of derivatives in equations (2.42) is three. Therefore $Y^{(e)}$ and $Z^{(e)}$ should have completeness of $(Y, \frac{dY}{d\xi}, \frac{d^2Y}{d\xi^2}, \frac{d^3Y}{d\xi^3})$ and $(Z, \frac{dZ}{d\xi}, \frac{d^2Z}{d\xi^2}, \frac{d^3Z}{d\xi^3})$ respectively. Thus N_i and B_i can be chosen identically. Interpolating functions satisfying these requirements are, Able and Desai [8]

$$\lfloor N_i(\xi) \rfloor = \lfloor B_i(\xi) \rfloor = \lfloor \xi_1^2 (3 - 2\xi), h\xi_1^2 \xi_2 \;\; \xi_2^2 (3 - 2\xi_2), -h \xi_1 \xi_2^2 \rfloor$$

where $\xi_1 = 1 - \frac{\xi}{h}$ and $\xi_2 = \frac{\xi}{h}$ (2.43)

Using equation (2.43), the equation (2.42a) becomes

$$\int_0^h \{N''\} \lfloor N'' \rfloor d\xi \, \{Y\}^{(ne)} - A_1 \int_0^h \{N'\} \lfloor N' \rfloor d\xi \, \{Y\}^{(ne)}$$

$$- A_2 \int_0^h \{N\} \lfloor N \rfloor d\xi \, \{Y\}^{(ne)} - \gamma x_j \int_0^h \{N'\} \lfloor N' \rfloor d\xi \{Y\}^{(ne)}$$

$$- \gamma \int_0^h \xi \{N'\} \lfloor N' \rfloor d\xi \{Y\}^{(ne)} + A_3 (1 - x_j) \int_0^h \{N'\} \lfloor N' \rfloor d\xi \{Z\}^{(ne)}$$

$$- A_3 \int_0^h \xi \{N'\} \lfloor N' \rfloor d\xi \{Z\}^{(ne)} - A_3 \int_0^h \{N\} \lfloor N' \rfloor d\xi \{Z\}^{(ne)}$$

$$+ A_4 \int_0^h \{N\} \lfloor N' \rfloor d\xi \{Z\}^{(ne)} - \frac{\alpha\ \Omega}{2} \int_0^h \{N''\} \lfloor N'' \rfloor d\xi \{Z\}^{(ne)}$$

$$- \frac{f\ \Omega}{2} \int_0^h \{N\} \lfloor N \rfloor d\xi \{Z\}^{(ne)} = \left\{ \begin{array}{c} \left.\frac{d^3Y^{(e)}}{d\xi^3}\right|_j \\ 0 \\ -\left.\frac{d^3Y^{(e)}}{d\xi^3}\right|_k \\ 0 \end{array} \right\} + \left\{ \begin{array}{c} 0 \\ -\left.\frac{d^2Y^{(e)}}{d\xi^2}\right|_j \\ 0 \\ \left.\frac{d^2Y^{(e)}}{d\xi^2}\right|_k \end{array} \right\}$$

$$+ A_1 \left\{ \begin{array}{c} \left.\frac{dY^{(e)}}{d\xi}\right|_j \\ 0 \\ -\left.\frac{dY^{(e)}}{d\xi}\right|_k \\ 0 \end{array} \right\} + A_3 \left\{ \begin{array}{c} -(1-x)\left.\frac{dZ^{(e)}}{d\xi}\right|_j \\ 0 \\ (1-x)\left.\frac{dZ^{(e)}}{d\xi}\right|_k \\ 0 \end{array} \right\}$$

$$+ \gamma \left\{ \begin{array}{c} -x\left.\frac{dY^{(e)}}{d\xi}\right|_j \\ 0 \\ x\left.\frac{dY^{(e)}}{d\xi}\right|_k \\ 0 \end{array} \right\} + \frac{\alpha\ \Omega}{2} \left\{ \begin{array}{c} -\left.\frac{d^3Z^{(e)}}{d\xi^3}\right|_j \\ 0 \\ \left.\frac{d^3Z^{(e)}}{d\xi^3}\right|_k \\ 0 \end{array} \right\} + \frac{\alpha\ \Omega}{2} \left\{ \begin{array}{c} 0 \\ \left.\frac{d^2Z^{(e)}}{d\xi^2}\right|_j \\ 0 \\ \left.\frac{d^2Z^{(e)}}{d\xi^2}\right|_k \end{array} \right\}$$

$$\ldots \quad (2.44a)$$

Similarly equation (2.42b) can be written as

$$\int_0^h \{N''\}\lfloor N''\rfloor \, d\xi \, \{Z\}^{(ne)} - A_5 \int_0^h \{N'\}\lfloor N'\rfloor \, d\xi \, \{Z\}^{(ne)}$$

$$- A_2 \int_0^h \{N\}\lfloor N\rfloor d\xi \, \{Z\}^{(ne)} - \gamma \, x_j \int_0^h \{N'\}\lfloor N'\rfloor \, d\xi \, \{Z\}^{(ne)}$$

$$- \gamma \int_0^h \xi \{N'\}\lfloor N'\rfloor \, d\xi \, \{Z\}^{(ne)} + A_3(1-x_j) \int_0^h \{N'\}\lfloor N'\rfloor d\xi \, \{Y\}^{(ne)}$$

$$- A_3 \int_0^h \xi \{N'\}\lfloor N'\rfloor \, d\xi \, \{Y\}^{(ne)} - A_3 \int_0^h \{N\}\lfloor N'\rfloor \, d\xi \, \{Y\}^{(ne)}$$

$$+ A_6 \int_0^h \{N\}\lfloor N'\rfloor \, d\xi \, \{Y\}^{(ne)} + \frac{\alpha \, \Omega}{2} \int_0^h \{N''\}\lfloor N''\rfloor \, d\xi \, \{Y\}^{(ne)}$$

$$+ \frac{f \, \Omega}{2} \int_0^h \{N\}\lfloor N\rfloor \, d\xi \, \{Y\}^{(ne)}$$

$$= \begin{Bmatrix} \left.\dfrac{d^3 Z^{(e)}}{d\xi^3}\right|_j \\ 0 \\ -\left.\dfrac{d^3 Z^{(e)}}{d\xi^3}\right|_k \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\left.\dfrac{d^2 Z^{(e)}}{d\xi^2}\right|_j \\ 0 \\ \left.\dfrac{d^2 Z^{(e)}}{d\xi}\right|_k \end{Bmatrix} + A_5 \begin{Bmatrix} \left.\dfrac{dZ^{(e)}}{d\xi}\right|_j \\ 0 \\ -\left.\dfrac{dZ^{(e)}}{d\xi}\right|_k \\ 0 \end{Bmatrix}$$

$$+ A_3 \begin{Bmatrix} -(1-x)\left.\dfrac{Y^{(e)}}{d\xi}\right|_j \\ 0 \\ +(1-x)\left.\dfrac{Y^{(e)}}{d\xi}\right|_k \\ 0 \end{Bmatrix} + \gamma \begin{Bmatrix} -x\left.\dfrac{Z^{(e)}}{d\xi}\right|_j \\ 0 \\ +x\left.\dfrac{Z^{(e)}}{\xi}\right|_k \\ 0 \end{Bmatrix}$$

$$+ \frac{\alpha\ \Omega}{2} \begin{Bmatrix} \left.\dfrac{d^3Y^{(e)}}{d\xi^3}\right|_j \\ 0 \\ -\left.\dfrac{d^3Y^{(e)}}{d\xi^3}\right|_k \\ 0 \end{Bmatrix} + \frac{\alpha\ \Omega}{2} \begin{Bmatrix} 0 \\ -\left.\dfrac{d^2Y^{(e)}}{d\xi^2}\right|_j \\ 0 \\ \left.\dfrac{d^2Y^{(e)}}{d\xi^2}\right|_k \end{Bmatrix} \qquad (2.44b)$$

Combining equations (2.44a) and (2.44b), one gets

$$\begin{bmatrix} [A]^{(e)} & [B]^{(e)} \\ [E]^{(e)} & [G]^{(e)} \end{bmatrix} \begin{Bmatrix} \{Y\}^{(ne)} \\ \{Z\}^{(ne)} \end{Bmatrix}$$

$$= \begin{Bmatrix} \begin{Bmatrix} \left.\dfrac{d^3Y^{(e)}}{d\xi^3}\right|_j \\ 0 \\ \left.\dfrac{d^3Y^{(e)}}{d\xi^3}\right|_k \\ 0 \end{Bmatrix} \\ \begin{Bmatrix} \left.\dfrac{d^3Z^{(e)}}{d\xi^3}\right|_j \\ 0 \\ \left.\dfrac{d^3Z^{(e)}}{d\xi^3}\right|_k \\ 0 \end{Bmatrix} \end{Bmatrix} + \begin{Bmatrix} \begin{Bmatrix} 0 \\ -\left.\dfrac{d^2Y^{(e)}}{d\xi^2}\right|_j \\ 0 \\ \left.\dfrac{d^2Y^{(e)}}{d\xi^2}\right|_k \end{Bmatrix} \\ \begin{Bmatrix} 0 \\ -\left.\dfrac{d^2Z^{(e)}}{d\xi^2}\right|_j \\ 0 \\ \left.\dfrac{d^2Z^{(e)}}{d\xi^2}\right|_k \end{Bmatrix} \end{Bmatrix}$$

$$
+ \left\{ \begin{matrix} A_1 \left\{ \begin{matrix} + \left.\dfrac{dY^{(e)}}{d\xi}\right|_j \\ 0 \\ - \left.\dfrac{dY^{(e)}}{d\xi}\right|_k \\ 0 \end{matrix} \right\} \\ \\ A_5 \left\{ \begin{matrix} - \left.\dfrac{dZ^{(e)}}{d\xi}\right|_j \\ 0 \\ \left.\dfrac{dZ^{(e)}}{d\xi}\right|_k \\ 0 \end{matrix} \right\} \end{matrix} \right\} + A_3 \left\{ \begin{matrix} \left\{ \begin{matrix} - (1-x) \left.\dfrac{dZ}{d\xi}\right|_j \\ 0 \\ (1-x) \left.\dfrac{dZ}{d\xi}\right|_k \\ 0 \end{matrix} \right\} \\ \\ \left\{ \begin{matrix} - (1-x) \left.\dfrac{dY}{d\xi}\right|_j \\ 0 \\ (1-x) \left.\dfrac{dY}{d\xi}\right|_k \\ 0 \end{matrix} \right\} \end{matrix} \right\}
$$

$$
+ \gamma \left\{ \begin{matrix} \left\{ \begin{matrix} -x \left.\dfrac{dY^{(e)}}{d\xi}\right|_j \\ 0 \\ x \left.\dfrac{dY^{(e)}}{d\xi}\right|_k \\ 0 \end{matrix} \right\} \\ \\ \left\{ \begin{matrix} -x \left.\dfrac{dZ}{d\xi}\right|_j \\ 0 \\ x \left.\dfrac{dZ^{(e)}}{d\xi}\right|_k \\ 0 \end{matrix} \right\} \end{matrix} \right\} + \frac{\alpha\ \Omega}{2} \left\{ \begin{matrix} \left\{ \begin{matrix} - \left.\dfrac{d^3Z^{(e)}}{d\xi}\right|_j \\ 0 \\ \left.\dfrac{d^3Z^{(e)}}{d\xi}\right|_k \\ 0 \end{matrix} \right\} \\ \\ \left\{ \begin{matrix} - \left.\dfrac{d^3Y^{(e)}}{d\xi^3}\right|_j \\ 0 \\ \left.\dfrac{d^3Y^{(e)}}{d\xi}\right|_k \\ 0 \end{matrix} \right\} \end{matrix} \right\}
$$

$$+\frac{\alpha\ \Omega}{2}\left\{\begin{matrix} \left\{\begin{matrix} 0 \\ +\left.\dfrac{d^2Z^{(e)}}{d\xi^2}\right|_j \\ 0 \\ -\left.\dfrac{d^2Z^{(e)}}{d\xi^2}\right|_k \end{matrix}\right\} \\ \left\{\begin{matrix} 0 \\ -\left.\dfrac{d^2Y^{(e)}}{d\xi^2}\right|_j \\ 0 \\ +\left.\dfrac{d^2Y^{(e)}}{d\xi}\right|_k \end{matrix}\right\} \end{matrix}\right\} \tag{2.45}$$

Matrices $[A]$, $[B]$, $[E]$ and $[G]$ are given in Appendix 3.

The equations (2.45) are for one element. Now assembling it for whole domain and applying boundary conditions, one gets

$$\begin{bmatrix} [A] & [B] \\ [E] & [G]_{BC} \end{bmatrix} \left\{\begin{matrix} \{Y\}^n \\ \{Z\}^n \end{matrix}\right\} = 0$$

or

$$[F]\left\{\begin{matrix} \{Y\}^n \\ \{Z\}^n \end{matrix}\right\} = 0 \tag{2.46}$$

Various types of boundary conditions are listed in Appendix 4.

2.3.2 Method of Solution:

Bolotin has shown that instability regions can be obtained by equating determinant of matrix [F] to zero. Hence frequencies Ω_j , j = 1, 2..... are the frequencies where Det [f] = 0 and they give boundaries of instability regions.

A Library subroutine FO3AAF/NAG was used to calculate the determinent of matrix [F]. The instabilities regions are plotted in $\Omega - \delta$ space.

CHAPTER - III

RESULTS AND DISCUSSION

This chapter deals with numerical results obtained for pipe conveying fluid for various types of pipe configurations. In Section 3.1 stabilities of pipes are discussed when the fluid is flowing at constant velocity. In Section 3.2 instability regions for various types of pipe configurations are plotted when a small harmonic component is superimposed over the mean velocity.

3.1 PIPES WITH STEADY FLOW

In this section critical velocities of various types of pipe configurations have been found by the finite element methods.

3.1.1 Number of Finite Element:

To begin with one must decide the number of finite elements to be used in the analysis. For every type of boundary condition the frequencies were calculated for few velocities with increasing number of finite elements. We selected that number of elements beyond which the increase in number of finite element did not change the value of frequency significantly.

Table 1 shows the frequencies of two equal span simply supported pipe with increasing number of finite elements for $\beta = 0.5$, $u = 2$ and $\alpha = \Gamma = p = \gamma = f = 0$. It is observed that the value does not change significantly after eight elements. So the number of finite elements taken for this case was eight.

3.1.2 Results and Discussions:

Using the finite element equations developed in Chapter-II the critical velocities for various configuration of pipes are obtained.

3.1.2.1 One Span Pipes

Complex frequencies for one span simply supported pipes were obtained by Paidoussis and Issid [21] for $\beta = 0.5$, $\alpha = \Gamma = p = \gamma = f = 0$. The same problem was studied by Deb [7] by using finite element method. However Deb [7] did not study the effect of internal dissipation (α) and gravity (γ) effects.

Figure 4 shows effect of coefficient of internal dissipation on the same system. The value of α is taken equal to 0.005. It is found that critical velocity does not change but the symmetry about imaginary axis is lost. These results match with those obtained by Paidoussis and Issid [21].

critical velocities of all these four cases are less than the critical velocity of three equal span pipe.

3.1.2.3 Multi-span Pipes, One End Fixed and Other Supports, Simple Supports

Critical velocity for two equal span pipe, case (1) Table 3, for $\beta = 0.5$, $\alpha = \Gamma = p = \gamma = f = 0$, is 7.12. This critical velocity is more than the critical velocity of pipe when all the supports are simple supports as expected. Next this pipe is considered with two unequal span lengths, cases (2, 3), Table 3. It is found when the intermediate support is shifted away from the fixed end critical velocity becomes 7.42 which is more than that of the pipe considered in case (1). On the other hand if the intermediate support is shifted towards the fixed end the critical velocity becomes 5.56 which is less than that of case (1).

Next three span pipes are considered, case (4, 5, 6,7), Table 3. It is found for equal span pipe the critical velocity is 9.93. Critical velocities for unequal spans, cases (5, 6, 7) are 8.60, 8.10 and 10.06 respectively. It may be noted that these critical velocities are more than those if all the supports are simple supports, see Table 2.

3.1.2.4 Multi-span Pipes with One End Fixed, Other End Free and Intermediate Supports Simple Supports

Critical velocity for this two equal span pipe, case (1), Table 4 is 4.64 for β = 0.5, $\alpha = \Gamma = p = \gamma = f = 0$. It the position of intermediate support is shifted towards the free end the critical velocity becomes 5.65, case (2), Table 4, On the other hand if intermediate support is shifted toward the fixed end, the critical velocity becomes 3.96 which is less than that of pipe considered in case (1).

Next three span pipes are considered, cases (4, 5, 6, 7) Table 4. The critical velocities for these cases are 6.48, 7.41, 5,79 and 5.24 respectively. It may be noted that these pipes are similar to these considered in cases (4, 5, 6, 7) of Table 3, except that the right hand support has been removed and is free now. It is observed that if the right hand support is free, the critical velocities decrease.

3.1.2.5 Multi-span Pipes with Both Ends Fixed and Supports Simple Supports

Critical velocity for two equal span pipe, case (1), Table 5, for β = 0.5, $\alpha = \Gamma = p = \gamma = f = 0$, is 9.0. For unequal span lengths case (2), Table 5 the critical velocity decreases to 8.65.

For three span pipes, cases (3, 4, 5, 6) critical velocities are 11.76, 9.90, 11.24 and 10.26 respectively. It should be noted that these pipes are similar to those considered in cases (4, 5, 6, 7) Table 3, except that the right hand support has been fixed. It is observed that in the present cases critical velocities increase.

3.1.2.6 Multi-span pipes with One End Fixed and Other Supports Elastic Supports

Table 6 gives the results for the pipes whose one end is fixed and all other supports are elastic supports. In cases (1, 2, 3, 4), Table 6 elastic supports are displacement springs. For $\beta = 0.5$, $\alpha = \Gamma = p = \gamma = f = 0$ and spring constant $\sigma_d = 10.0$ the critical velocities are 8.52, 7.14, 7.08 and 6.93 respectively. These velocities are less than that of those considered in cases (4, 5, 6, 7), Table 3. Where all the supports are simple supports as expected.

Next pipes with torsional springs are considered, see cases (5, 6,7, 8), Table 6. The critical velocities are 13.41, 10.92, 10.71 and 10.38 respectively. These velocities are more than that of these considered in cases (4, 5, 6, 7), Table 3 as expected.

3.1.2.7 Multi-span Vertical Pipes

Table 7 gives the critical velocities for vertical multi-span pipes. It is observed that critical velocities are more in case of downward flow than those for horizontal flow, and critical velocities are less in case of upward flow. It is observed that effect of gravity is more pronounced in multi-span pipes if one of the ends is free.

3.2 PIPES WITH HARMONICALLY PERTURBED FLOW

In this section we will discuss the stability of pipes when a small harmonic component is superimposed over the mean velocity.

3.2.1 Results and Discussions:

Using the finite element equations developed in Chapter-II the regions of principal primary instability are obtained for various types of pipe configurations. These regions are plotted in Ω - δ space.

3.2.1.1 One Span Pipes

In Figure 5 instability regions for one span simply supported pipe are plotted for $u = 0.6\pi$, $\alpha = \Gamma = p = \gamma = f = 0$ and $\beta^{1/2} = 0.4$ and $\beta^{1/2} = 0.8$ These results match with those obtained in [21,13].

For $\beta^{1/2} = 0.8$ results are also given in Table 8 and compared with Kulkarni [13]. Excellent matching is observed. It is observed if the value of β is increased instability region broadens.

Results for fixed-fixed pipe are also given in the Figure 5. Instability regions are plotted for $\beta^{1/2} = 0.4$, $\alpha = \Gamma = p = \gamma = f = 0$, $u = 0.6\ \pi$ and π. It is observed that with increase in velocity u the instability region shifts to lower value of Ω.

In Figure 6 instability regions are plotted for fixed simply supported pipe for $\beta^{1/2} = 0.4$, $\alpha =$ $= p = \gamma = f = 0$, $u = 0.6\ \pi$ and π. Again it was observed that with increase in value of u instability region shifts to lower value of Ω. In the same figure instability regions for cantilever pipe are also plotted for $\beta^{1/2} = 0.4$, $\alpha = \Gamma = p = \gamma = f = 0$, $u = \pi$ and $u = 1.2\ \pi$. A striking feature of this instability region is that it starts after a finite value of δ. Again instability region shifts itself to lower value of Ω for higher velocity.

3.2.1.2 Multi-span Simply Supported Pipes

Instability regions associated with first two modes of two equal span simply supported pipe for $u = 1.2\ \pi$, $\beta^{1/2} = 0.8$, $\alpha = \Gamma = p = \gamma = f = 0$ are

given in Figure 7 and Table 9. The results obtained by Kulkarni [13] are also given in the same figure and table. By present analysis we obtained broader instability regions. The difference arises due to the change in the governing differential equation of motion. Kulkarni [13] has taken L in third term in equation (2.1) as length of the span, where actually L is the overall length of the pipe.

Figure 8 and Figure 9 give the instability regions associated with first two modes of two and three span (equal and unequal) simply supported pipe for $\beta^{1/2} = 0.4$, $\alpha = \Gamma = p = \gamma = f = 0$, respectively. For two span pipe velocity u is $0.6\ \pi$ and for three span pipe it is $1.8\ \pi$. It is seen that for unequal span lengths the instability region associated with first mode shifts to lower value of Ω in comparison to first mode of equal span pipes. On the other hand the instability region corresponding to second mode shifts to higher value of Ω in comparison to instability region corresponding to second mode of equal span pipes. The instability region corresponding to first mode of one span pipe is also plotted in Figure 8. It can be seen that for one span pipe the instability region are at very low value of Ω.

3.2.1.3 Multi-span Pipes, one End Fixed and Other supports Simple Supports

Figure 10 and Figure 11 show the instability regions associated with first two modes of two and three span (equal and unequal) pipe for $\beta^{1/2} = 0.4$, $\alpha = \Gamma = p = \gamma = f = 0$ respectively. For two span pipe u is 1.2 π and for three span pipe it is 1.8 π. It is observed that regions of instability associated with first mode of unequal spans are at lower value of Ω in comparison to that of equal spans. On the other hand the instability regions associated with second mode are at higher value of Ω in case of unequal spans. In Figure 10 instability regions corresponding to first mode of one span fixed - simply supported are also plotted. It is observed that for one span pipe instability region is at very low value of Ω.

3.2.1.4 Multi-span Pipes, Both Ends Fixed and Other Supports Simple Supports

Figure 12 gives instability regions associated with first two modes of a two span (equal and unequal) pipe for $u = 1.2\ \pi$, $\beta^{1/2} = 0.4$, $\alpha = \Gamma = p = \gamma = f = 0$. The region associated with first mode is at lower value of Ω for unequal span length. But region of

instability associated with second mode is at higher value of Ω for unequal spans. The results for one span are also plotted for comparison purposes.

Figure 13 gives instability regions associated with first two modes of three span (equal and unequal) pipe for $u = 1.8\ \pi$, $\beta^{1/2} = 0.4$, $\alpha = \Gamma = p = \gamma = f = 0$. Here the instability regions associated with both the modes are at lower value of Ω for unequal spans.

3.2.1.5 Multi-span Pipes, One End Fixed, Other and Free and Intermediate Supports Simple Supports

Figure 14 shows the instability regions for a two span (equal and unequal) pipe. The interesting feature is that the instability region starts from a finite value of δ and both modes are at higher value of Ω for unequal spans.

Figure 15 gives instability regions for a three span pipe for $u = 1.8\ \pi$, $\beta^{1/2} = 0.4$, $\alpha = \Gamma = p = \gamma = f = 0$. The instability regions of both modes are at lower value of Ω for unequal spans. The instability region corresponding to the second mode brodens in case of unequal spans.

CHAPTER IV

CONCLUSIONS

Based on the results obtained in Chapter III following conclusions are drawn.

1. The results by finite element method for multi-equispan simply supported pipes with steady flow match with those obtained by earlier authors. This shows that finite element equations of motion are correct and can be applied for multi-span (equal and unequal) pipes with any type of supports.

2. The results by the finite elements method for multi-equispan simply supported pipes with harmonically perturbed flow match with those obtained by earlier authors for $\beta = 0$. For $\beta \neq 0$ results differ from earlier results because of correct equation of motion used here. Thus the finite element equations of motion developed here are correct and can be applied for multi-span (equal and unequal) pipes with any type of supports.

3. The method developed is very general and flexible. The critical velocities and instability regions for any type of pipe configurations can be obtained by changing the input data in the computer program.

4. The column matrices $\left\{ N(u^2 - \Gamma + p(1-2\nu\mu) - \gamma) \frac{\partial w}{\partial \xi} \Big|_0^h \right\}$ and $\left\{ \nu N(x_j + \xi)\frac{\partial w}{\partial \xi} \Big|_0^h \right\}$ of equation (2.11) should be transferred to square matrix of left hand side, similarly. third, fourth fifth column matrics on right hand side of equation (2.45) should be transferred to square matrices of left hand sides,as suggested by Deb [7]. These column matrices can not be neglected as in suggested by the work of Szabo and Lee [24].

5. Critical velocities of pipes can be controlled by changing the position of the supports. Critical velocities of vertical pipes decrease for upward flow and increase for downward flow in comparison to horizontal flow. This effect is more pronounced if one end is free.

6. The principal primary instability regions broadens when the value of β is increased. The principal primary instability regions shifts to lower value of Ω for increase in velocity u.

7. If one end of the pipe is free the principal primary instability regions starts from a finite value of δ''.

8. Instability regions can be controlled by changing the position of the supports.

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TABLE 1

Number of elements vs Frequencies

Two equal span simply supported pipe

$\beta = 0.5, \ \alpha = \Gamma = p = \gamma = f = 0, \ u = 2.0$

Number of Elements	First Mode		Second Mode	
	Real	Imaginary	Real	Imaginary
4	0.0	5.858	0.0	22.798
6	0.0	5.626	0.0	22.310
8	0.0	5.584	0.0	22.194
10	0.0	5.576	0.0	22.188
12	0.0	5.562	0.0	22.186

TABLE 2

<u>Critical velocities for Multi span simply supported pipes</u>

$\beta = 0.5,\ \alpha = \Gamma = p = \gamma = f = 0$

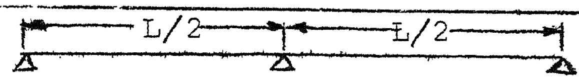
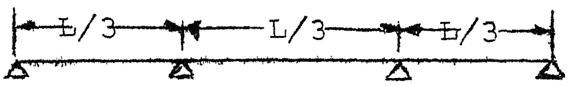
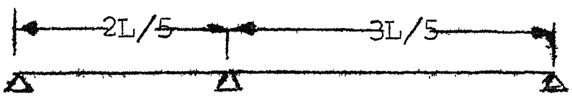
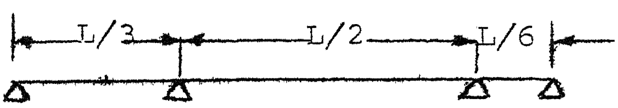
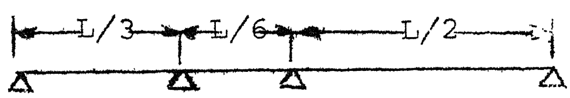
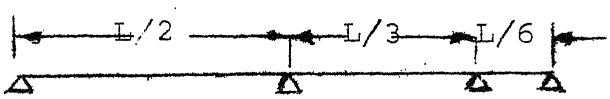
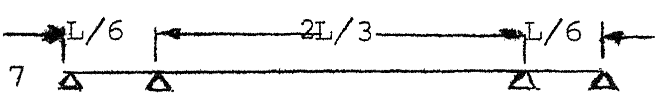
	Type of pipe	Number of Element	Critical Velocity u_c
1		8	2π
2		12	$3.\pi$
3		8	6.12
4		12	8.52
5		12	8.16
6		12	8.01
7		12	7.98

TABLE 3

Critical velocities of Multi-span pipes - one end fixed and other supports simple support

$\beta = 0.5, \; \alpha = \Gamma = p = \gamma = f = 0$

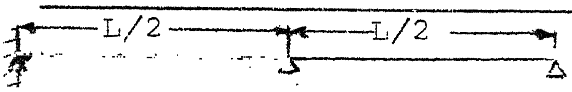
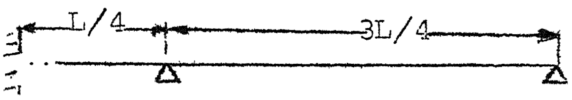
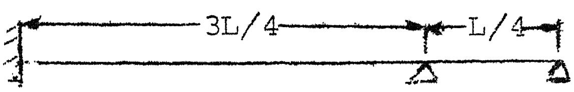
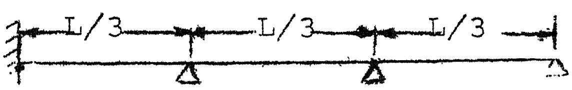
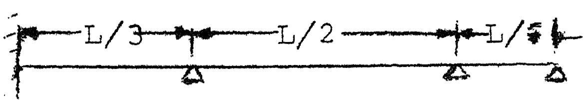
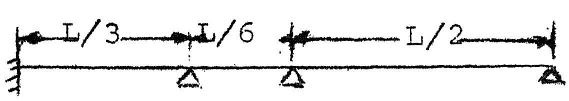
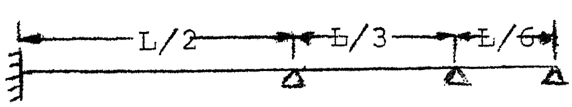
	Type of pipe	Number of Element	Critical Velocity
1		8	7.12
2		8	5.56
3		8	7.42
4		12	9.93
5		12	9.60
6		12	8.10
7		12	10.06

TABLE 5

Critical velocities for Multi-span pipes with both end fixed and other supports simple supports

$$\beta = 0.5, \quad \alpha = \Gamma = p = \gamma = f = 0$$

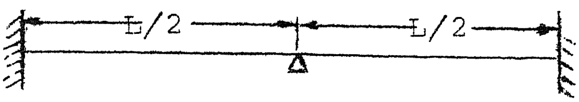
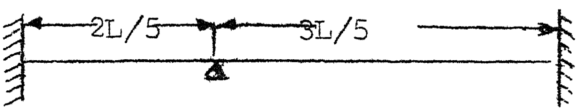
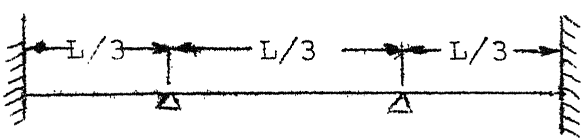
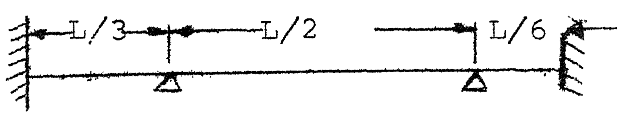
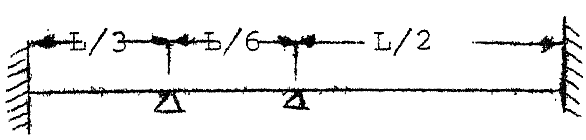
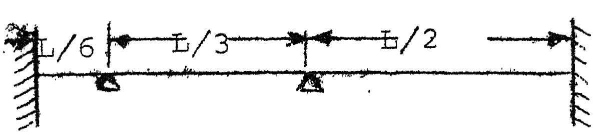
Type of pipe	Number of Element	Critical Velocity
1 	8	9.00
2 	8	8.65
3 	12	11.76
4 	12	9.90
5 	12	11.24
6 	12	10.26

TABLE 4

Critical velocities for Multi-span pipes with one end fixed, other end free and intermediate supports simple supports

$\beta = 0.5, \quad \alpha = \Gamma' = p = \gamma = f = 0$

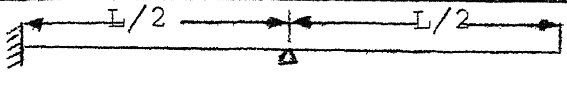
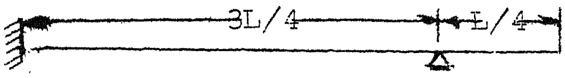
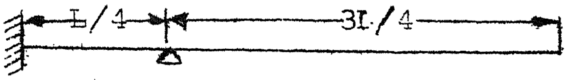
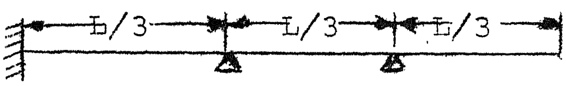
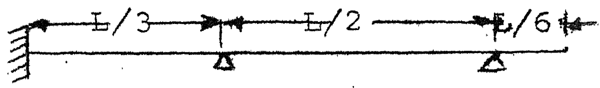
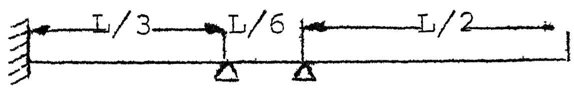
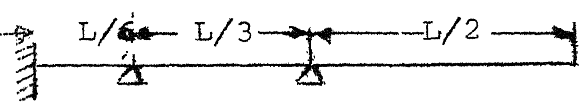
Type of Pipe	Number of Element	Critical Velocity
1 	8	4.64
2 	8	5.65
3 	8	3.96
4 	12	6.48
5 	12	7.41
6 	12	5.79
7 	12	5.24

TABLE 6

Critical velocities for Multiple span pipes with one end fixed and other supports elastic supports

$\beta = 0.5, \ \alpha = \Gamma = p = \gamma = f = 0$

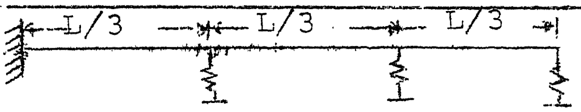
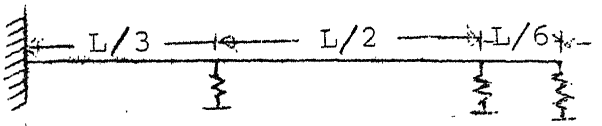
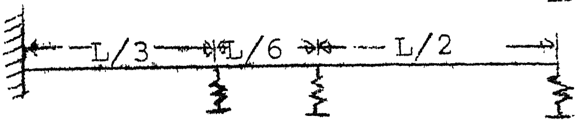
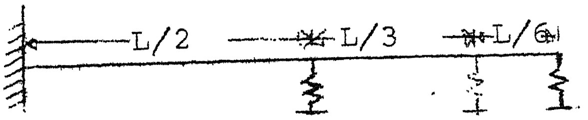
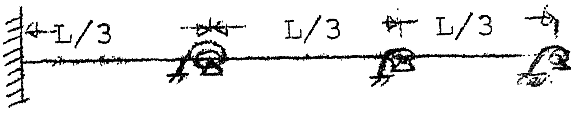
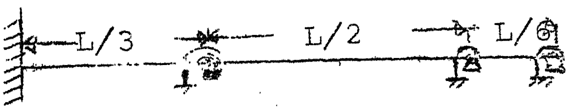
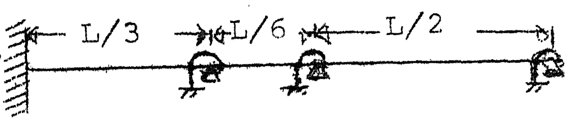
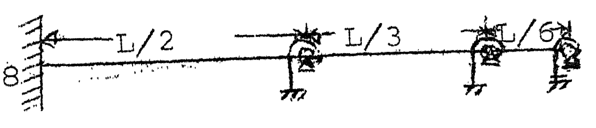
Type of pipe	Spring constant	Number of Elements	Critical Velocity
1 	α_d=10.0	12	8.52
2 	α_d=10.0	12	7.14
3 	α_d=10.0	12	7.08
4 	α_d=10.0	12	6.93
5 	α_t= 5.0	12	13.41
6 	α_t= 5.0	12	10.92
7 	α_t= 5.0	12	10.71
8 	α_t= 5.0	12	10.38

TABLE 7

Critical velocities for Multi-span vertical pipes

$\beta = 0.5,\ \alpha = \Gamma = p = \gamma = f = 0$

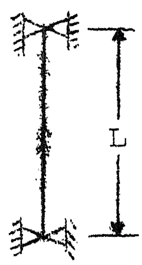
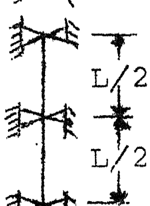
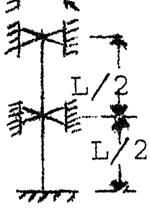
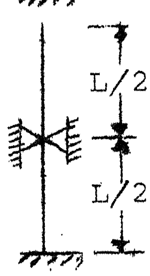
Type of pipe		Number of Element			
			$\gamma = -5$	$\gamma = 0$	$\gamma = 5$
1		5	2.71	π	3.51
2		8	6.05	2π	6.46
3		8	7.03	7.12	7.23
4		8	4.26	4.64	5.08

TABLE 8

Values of frequencies bounding principal primary instability region for single span simply supported pipe

$u = 0.6\ \pi,\ \beta^{1/2} = 0.8,\ \alpha = \eta = p = \gamma = f = 0$

NEL = 5

	Frequencies	
	Lower	Upper
0.01	15.345	15.507
0.10	14.613	16.204
0.20	13.765	16.948
0.30	12.883	17.654
0.40	11.955	18.319

TABLE 9

Values of frequencies bounding the principal primary instability regions associated with first two modes of a two span pipe obtained by present analysis and by Kulkarni [13]

$$u = 1.2\pi, \ \beta^{1/2} = 0.8, \ \alpha = \Gamma = p = \gamma \ f = 0$$

NEL = 8

	First Mode				Second Mode			
	Lower		Upper		Lower		Upper	
	Pre- sent	Kul- karni [13]	Pre- sent	Kul- karni [13]	Pre- sent	Kul- karni [13]	Pre- senr	Kul- karni [13]
0.01	61.20	61.40	62.29	62.40	109.60	109.88	110.92	110.44
0.10	56.07	58.52	67.17	65.72	104.56	106.08	116.67	113.06
0.20	50.38	55.02	71.97	66.56	99.69	104.04	123.84	116.08
0.30	44.87	51.60	75.59	71.12	96.02	101.00	131.50	118.84
0.40	39.60	47.88	77.10	72.36	94.21	98.16	139.24	131.48

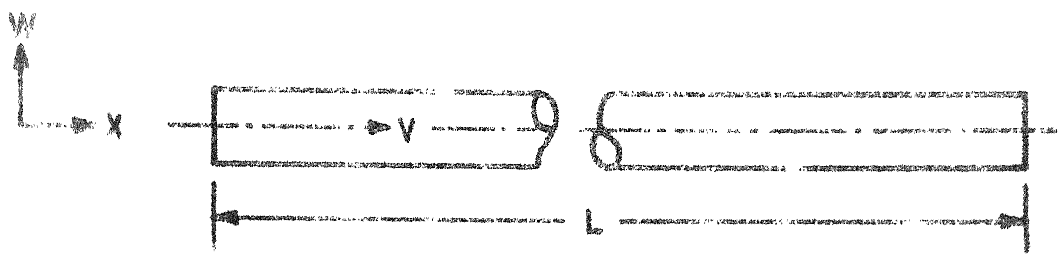


Fig 1 Pipe conveying fluid at velocity V

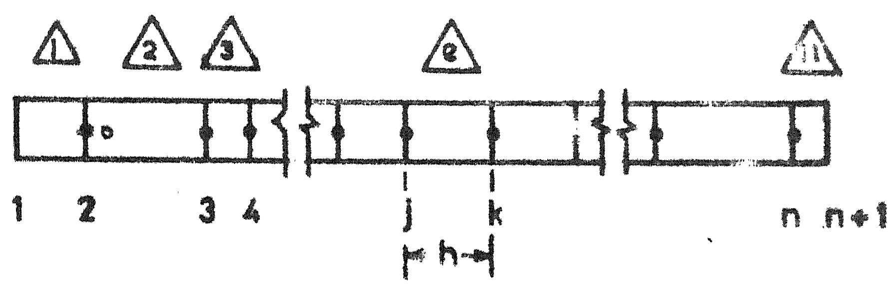


Fig 2 Finite elements of the pipe

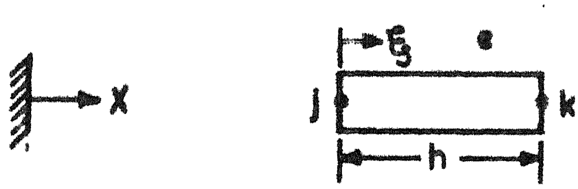


Fig. 3 Typical finite element of the pipe

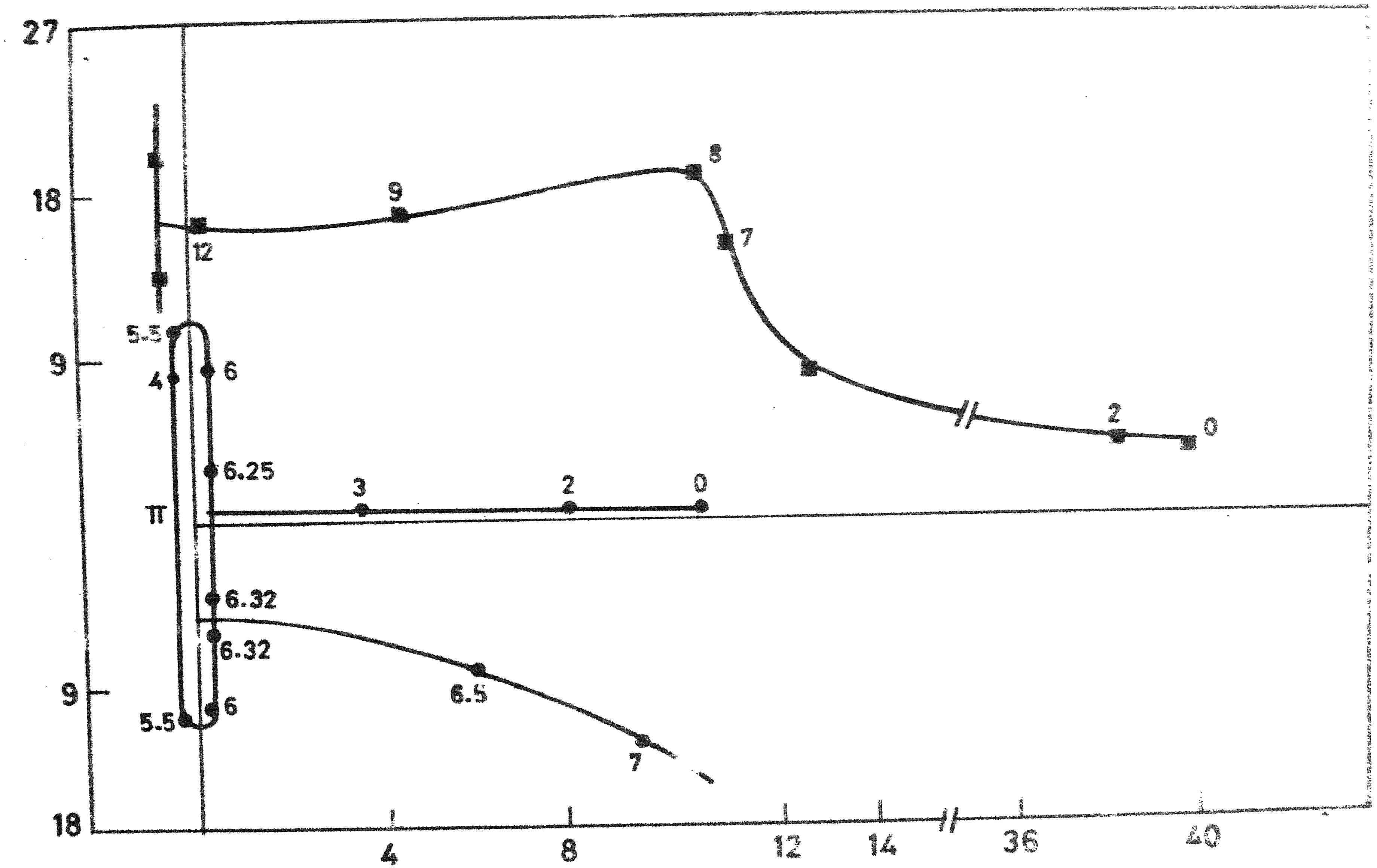


FIG. 4 Dimensionless complex frequency diagram of a simply supported pipe
$\beta^{1/2} = 0.5$ $\alpha = 0.005$ and $p = \Gamma = \nu = f = 0$ NEL = 5
—●— First Mode. —■— Second Mode.

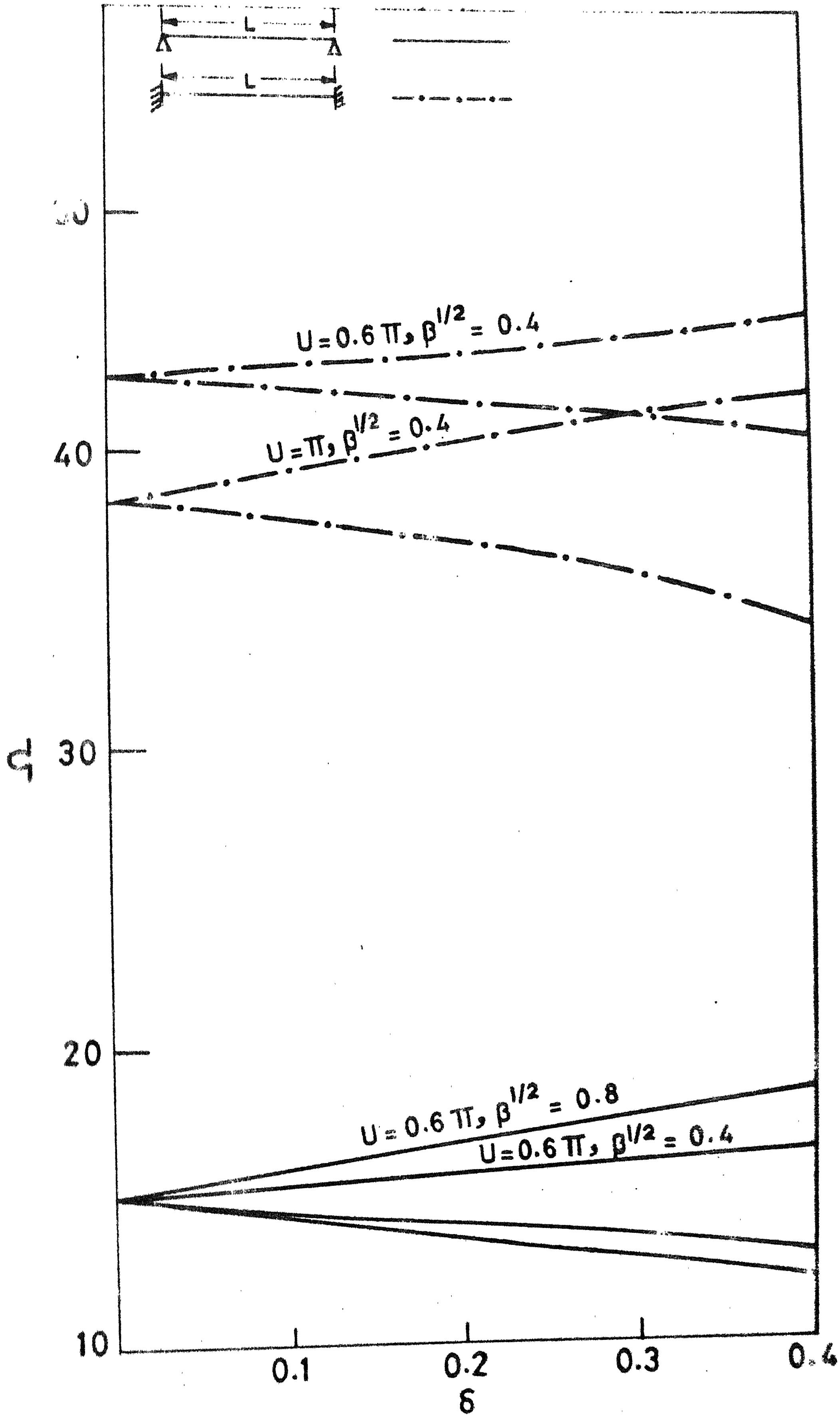


Fig.5 Regions of principal primary instability of one span pipes for
$\alpha = \Gamma = p = \gamma = f = 0$ NEL = 5

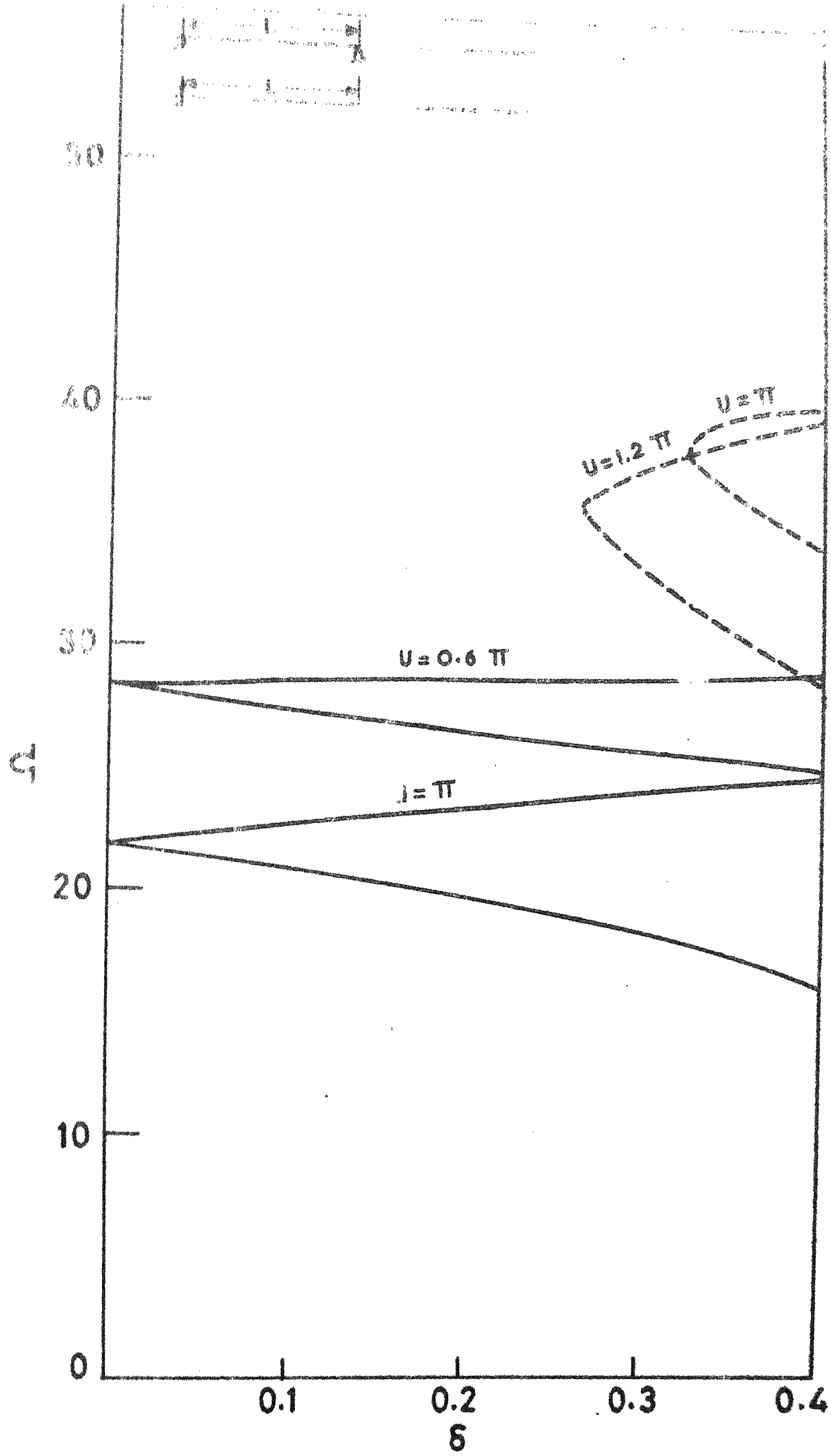


Fig.6 Regions of principal primary instability of one span pipes for

$\beta^{1/2} = 0.4, \alpha = \Gamma = p = \gamma = f = 0$ NEL=5

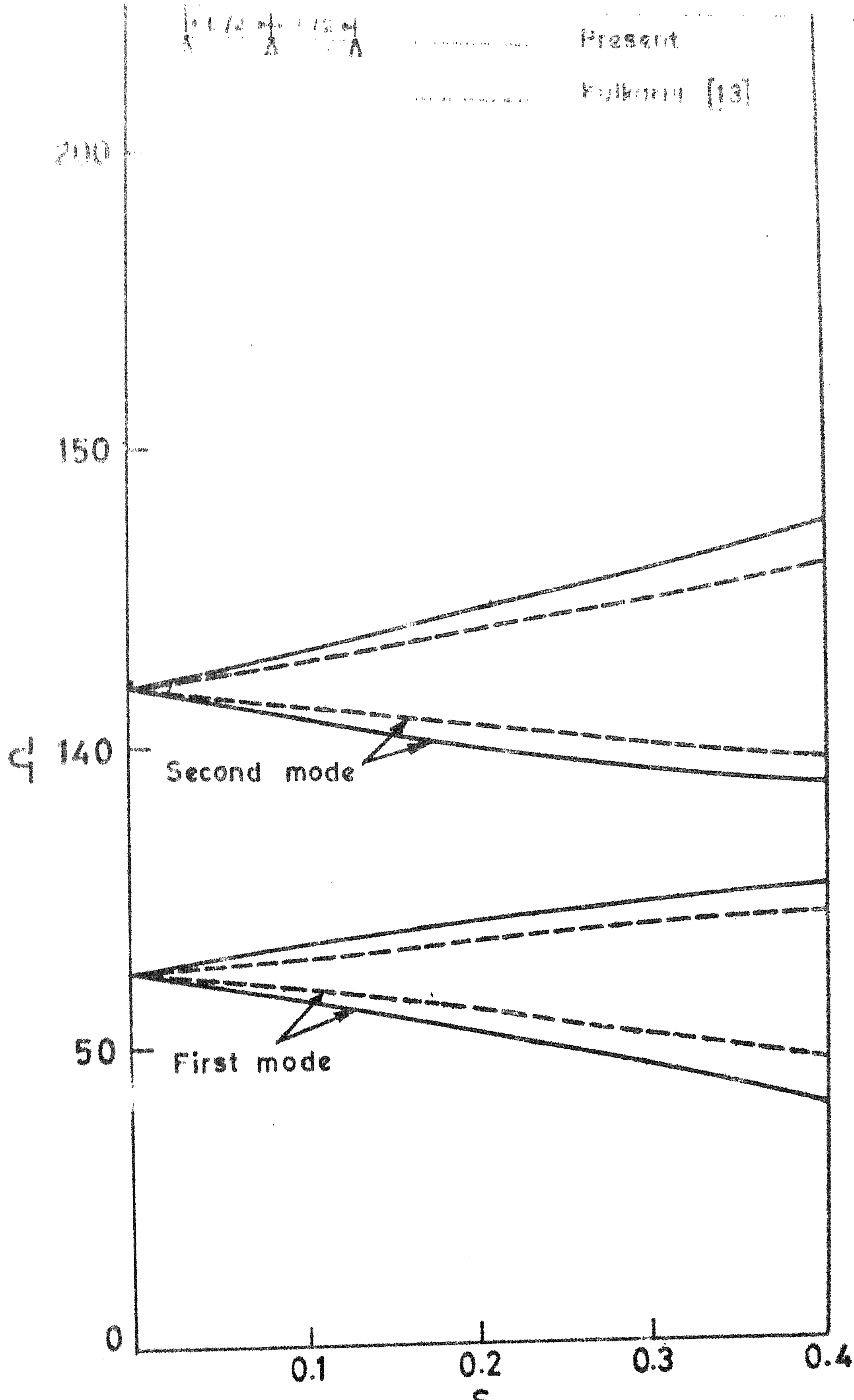


Fig.7 Regions of principal primary instability of two span simply supported pipe for $u=1.2\pi$, $\beta^{1/2}=0.8$, $\alpha=\Gamma=p=\gamma=f=0$ NEL = 8

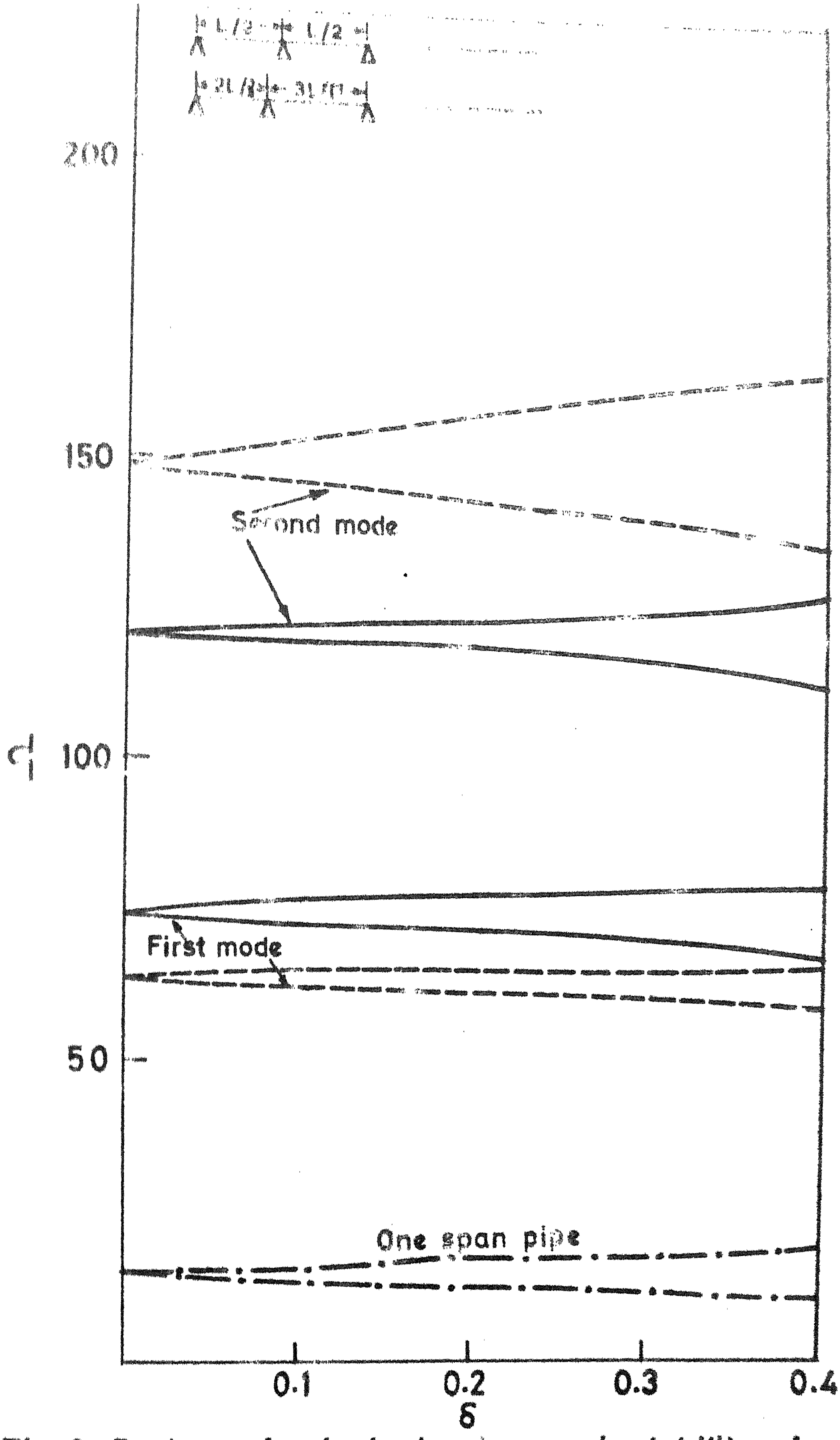


Fig. 8 Regions of principal primary instability of a two span simply supported pipes for $u = 0.6\pi, \beta^{1/2} = 0.4, \alpha = \Gamma = p = \gamma = f = 0$ NEL = 8

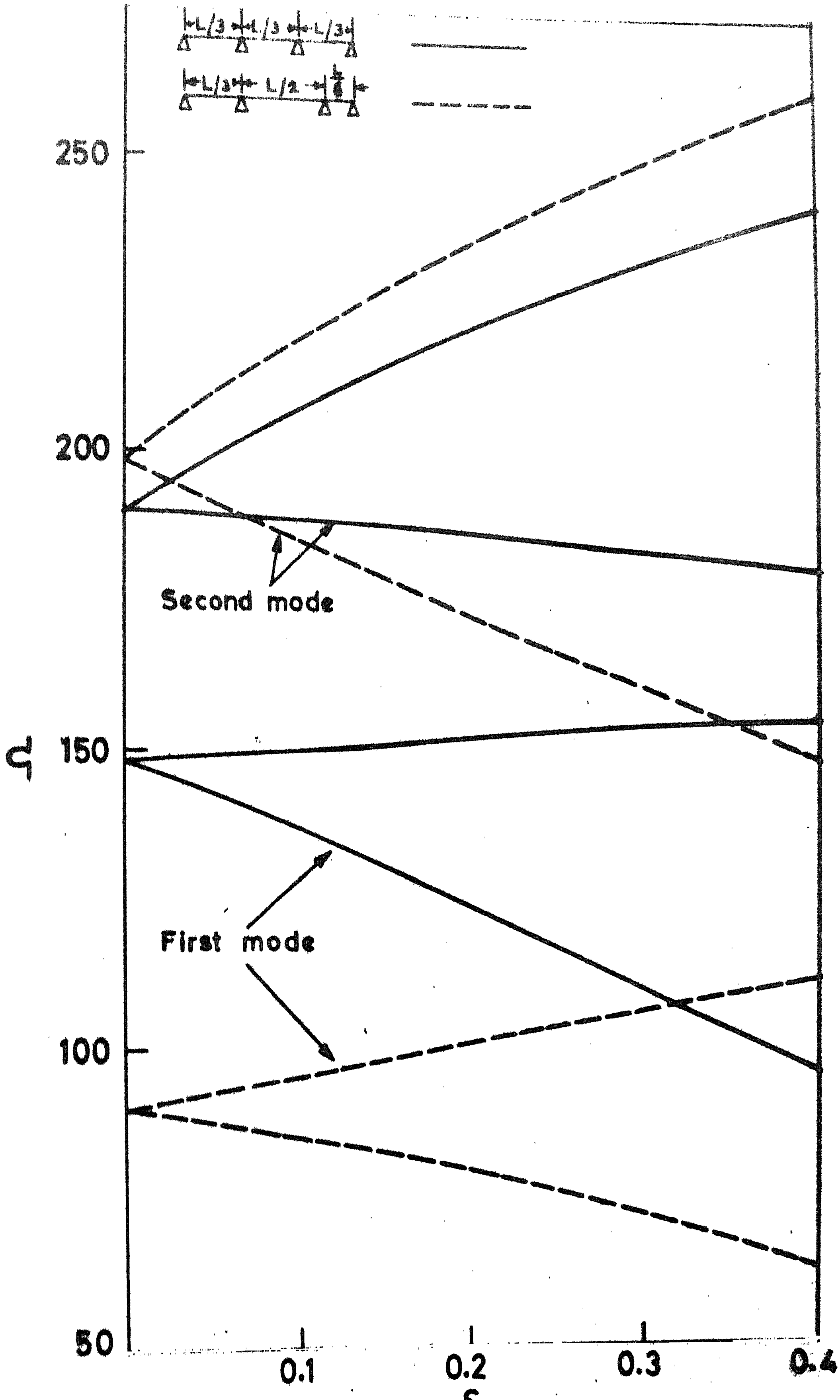


Fig. 9 Regions of principal primary instability of three span simply supported pipes for

$u = 1.8\,\pi,\ \beta^{1/2} = 0.4,\ \alpha = \Gamma = p = \gamma = f = 0$ NEL = 12

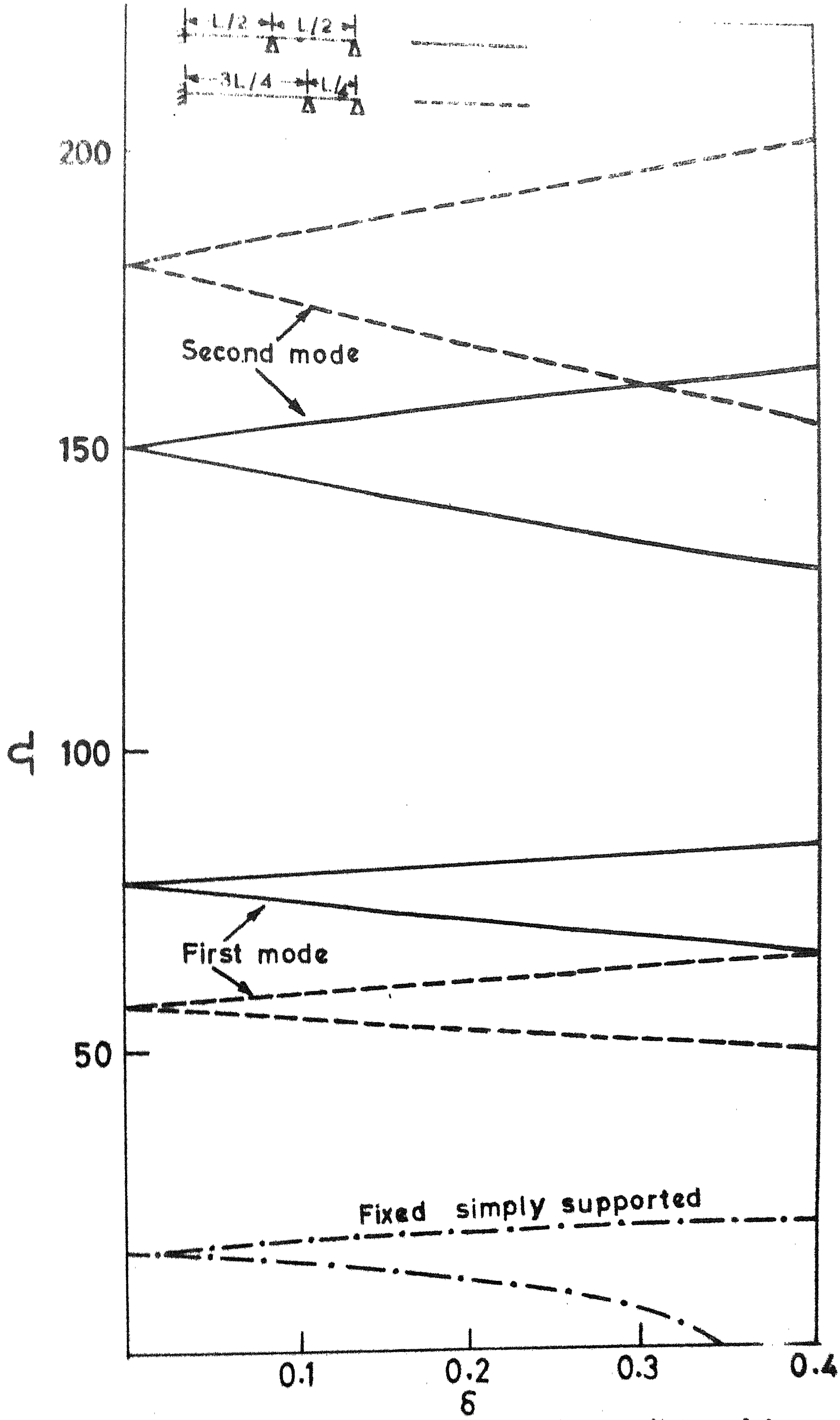


Fig.10 Regions of principal primary instability of two span pipes one end fixed and other supports simple supports for

$u = 1.2\pi,\ \beta^{1/2} = 0.4,\ \alpha = \Gamma = p = \gamma = f = 0$ NEL = 8

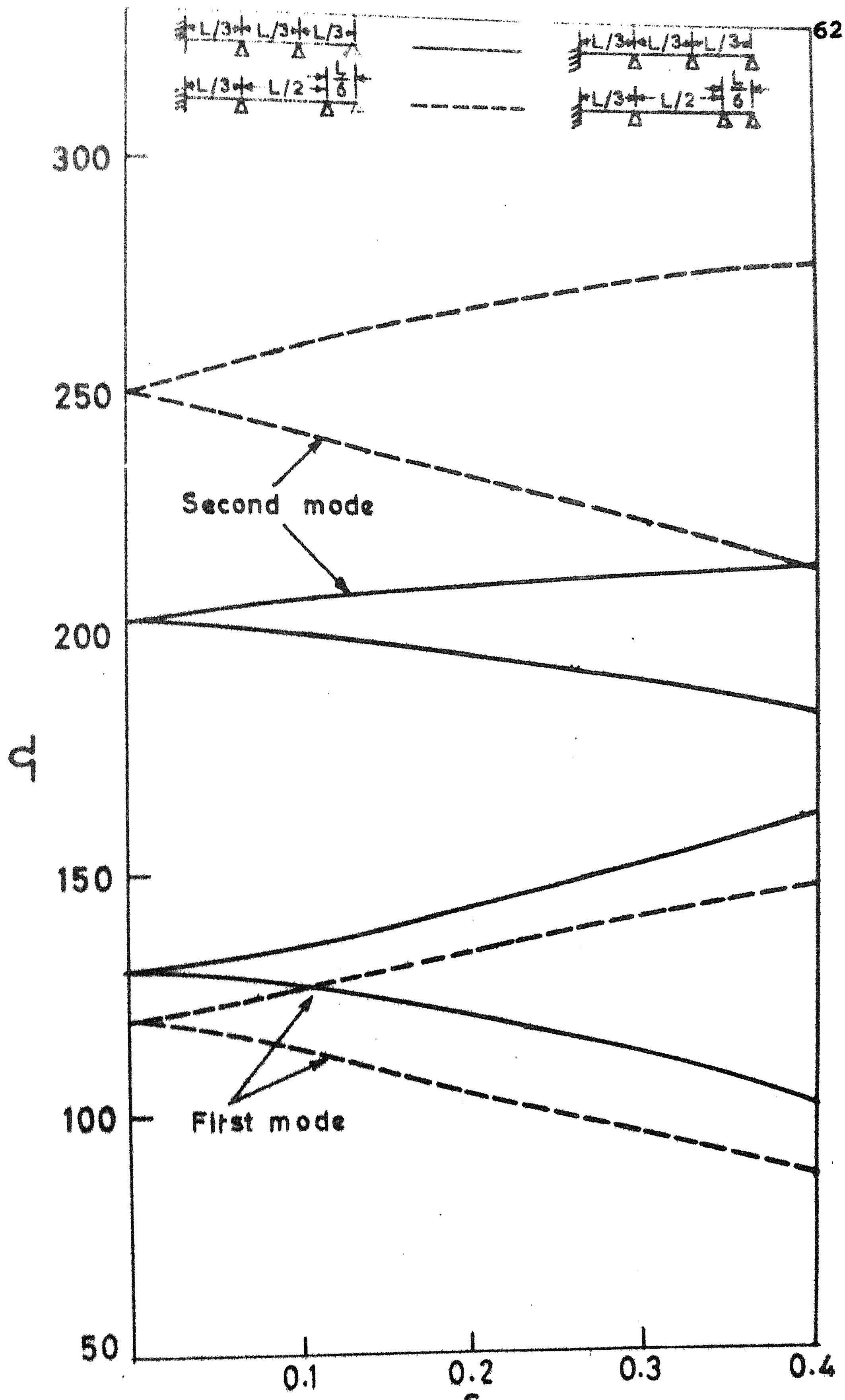


Fig.11 Regions of principal primary instability of three span pipes one end fixed and other supports simple supports for
$u = 1.8\pi$, $\beta^{1/2} = 0.4$, $\alpha = \Gamma = p = \gamma = f = 0$ NEL=12

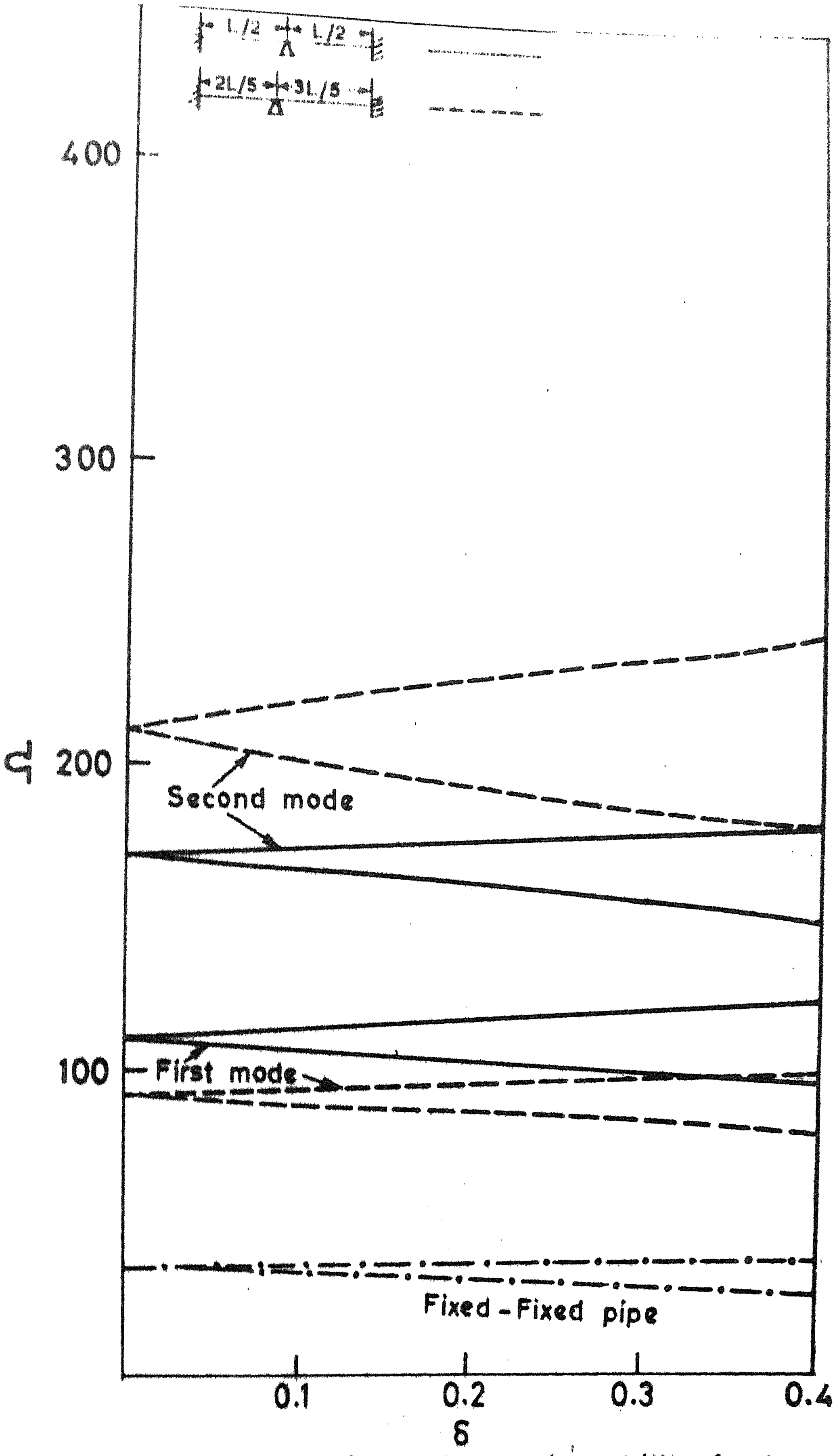


Fig.12 Regions of principal primary instability for two span pipes both ends fixed and other support simple support for
$u = 1.2\pi$, $\beta^{1/2} = 0.4$, $\alpha = \Gamma = p = \gamma = f = 0$ NEL = 8

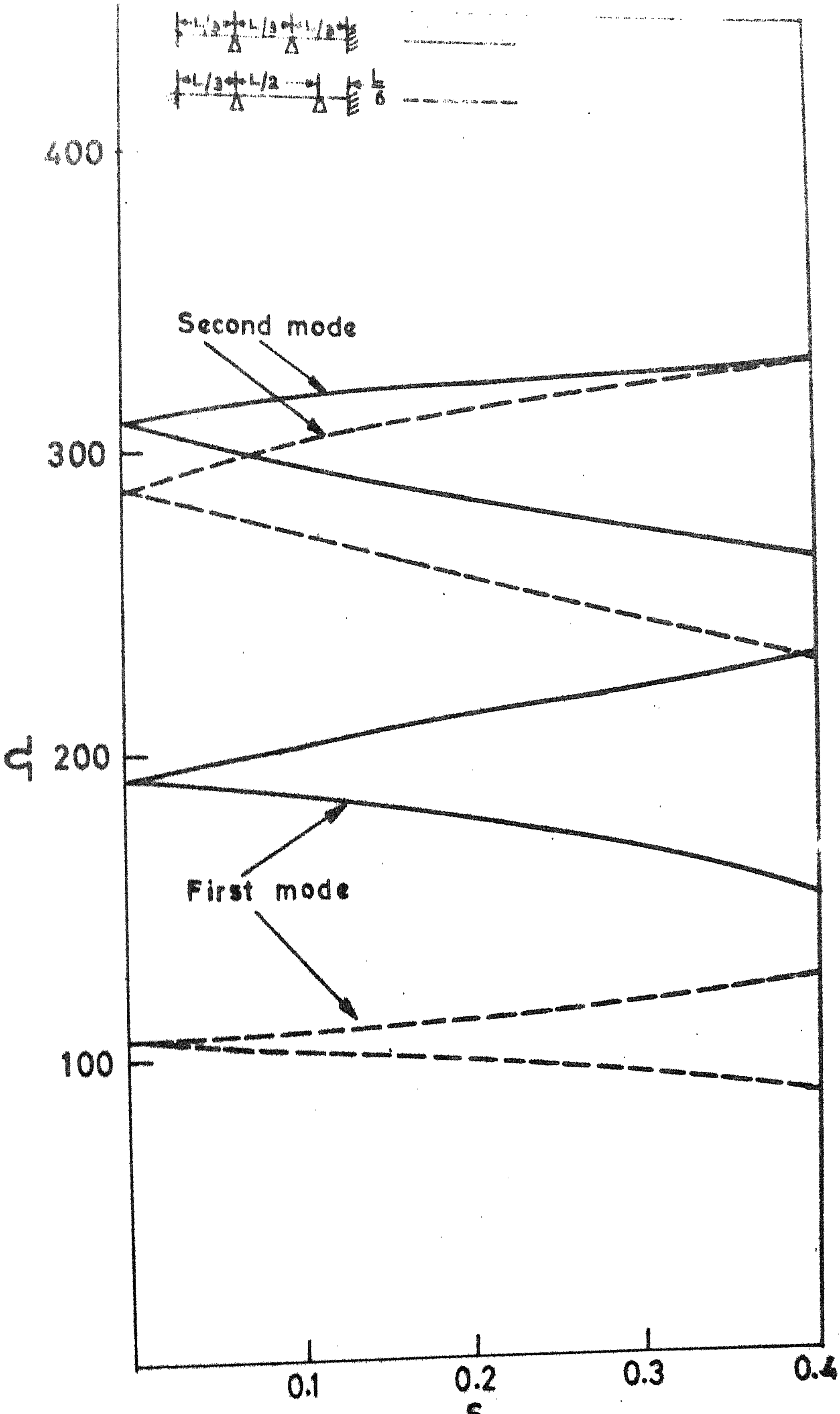


Fig.13 Regions of pincipal primary instability of three span pipes both ends fixed and other supports simple supports

$u = 1.8\pi$, $\beta^{1/2} = 0.4$, $\alpha = \Gamma = p = \gamma = f = 0$ NEL = 12

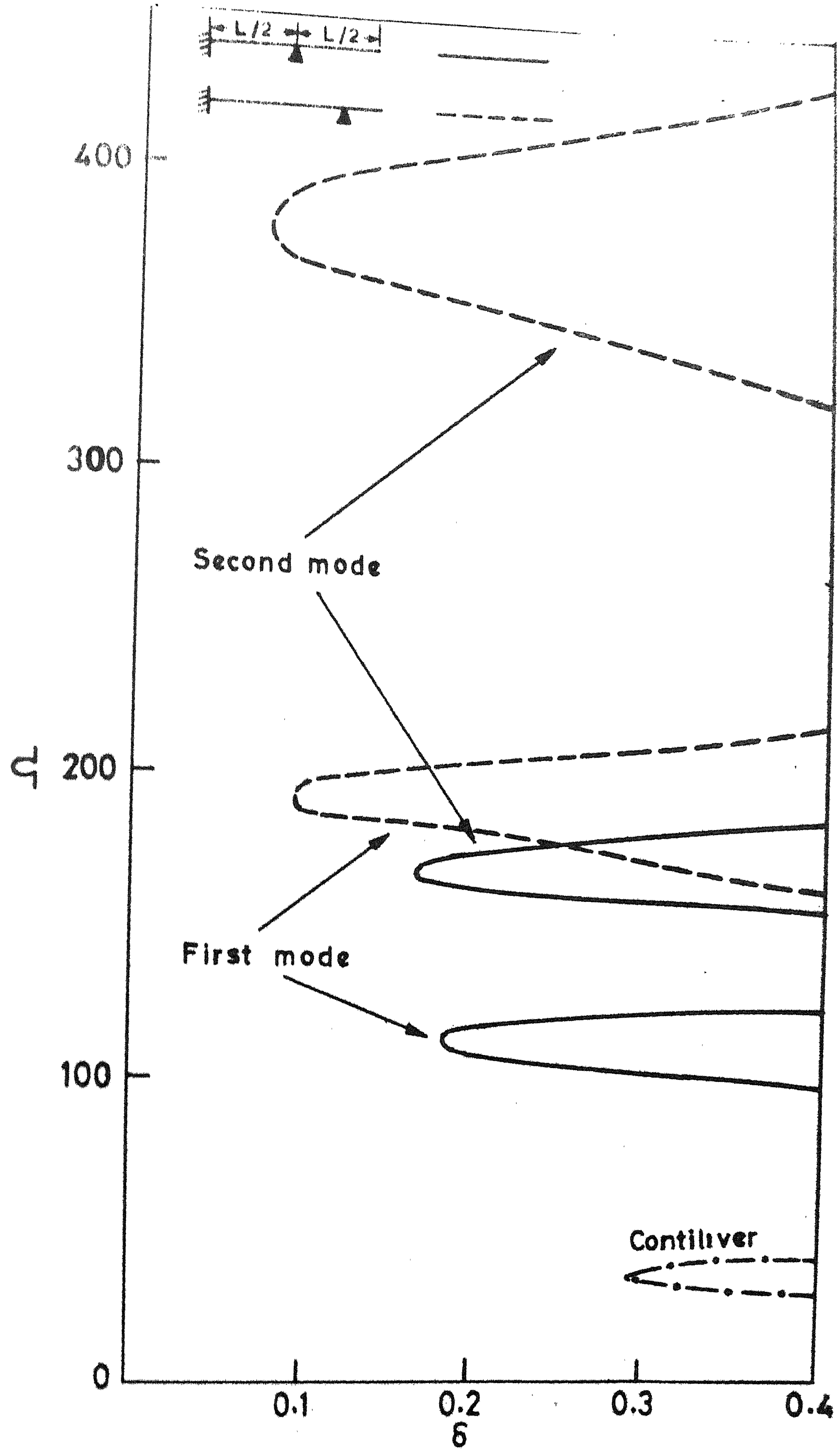


Fig.14 Regions of principal primary instability of two span pipes one end fixed other free and intermediate support simple suppot for $u = 1.2\pi, \beta^{1/2} = 0.4, \alpha = \Gamma = p = \gamma = f = 0$ NEL = 8

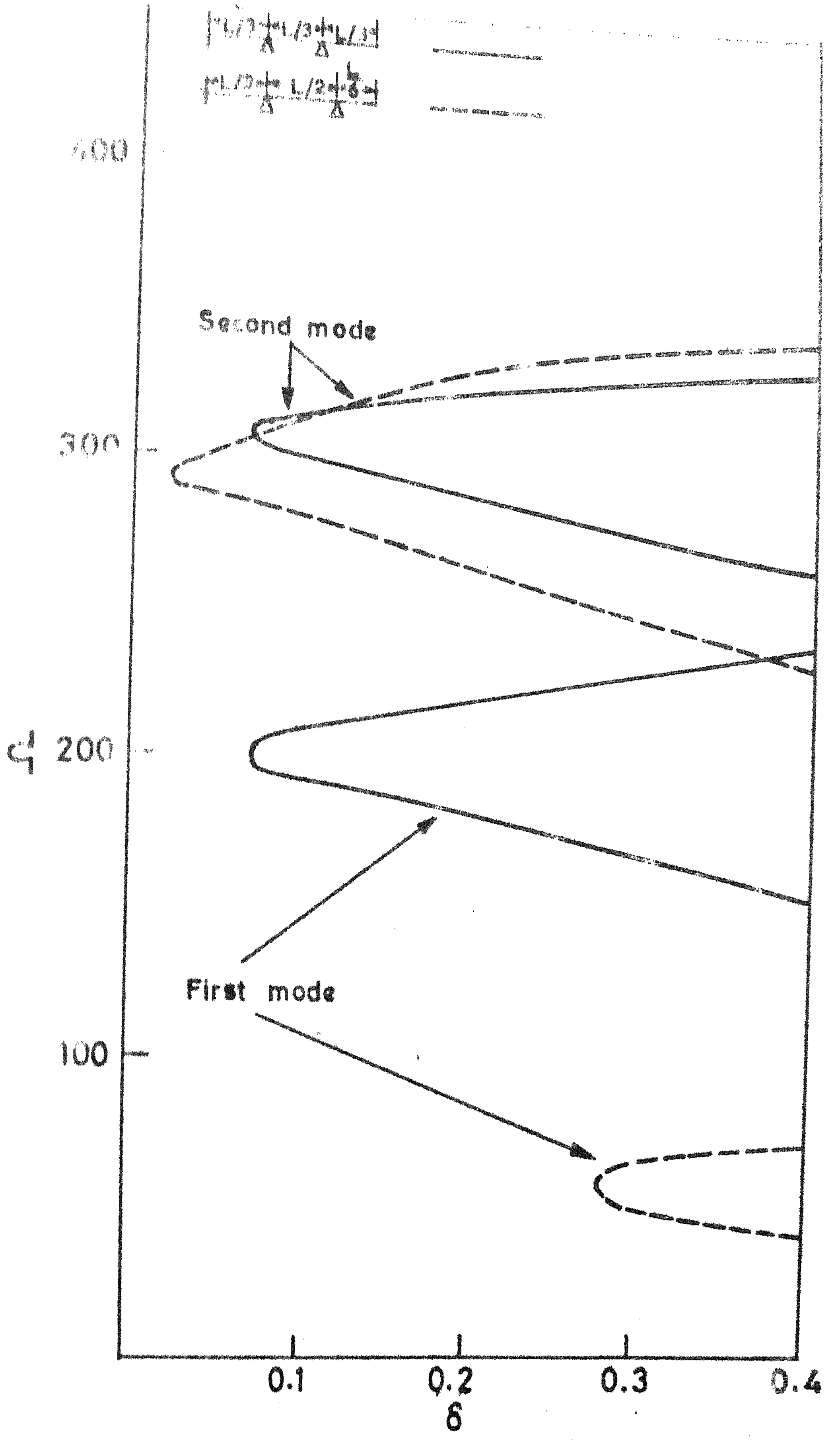


Fig.15 Regions of principal primary instability of three span pipes one end fixed other free and intermediate supports simple supports for $u=1.8\pi$, $\beta^{1/2}=0.4$, $\alpha=\Gamma=p=\gamma=f=0$ NEL=12

APPENDIX-1

The values of various matrices are as follows

$$[m]^e = \frac{h}{420}\begin{bmatrix} 156 & 22h & 54 & -13h \\ & 4h^2 & 13h & -3h^2 \\ & & 156 & -22h \\ & \text{sym} & & 4h^2 \end{bmatrix} \qquad (A1.1)$$

$$[c]^{(e)} = \frac{\alpha}{h^3}\begin{bmatrix} 12 & 6h & -12 & 6h \\ & 4h^2 & -6h & 2h^2 \\ & & 12 & -6h \\ \text{sym} & & & 4h^2 \end{bmatrix} + \frac{2\beta^{1/2}u}{60}\begin{bmatrix} -30 & 6h & 30 & -6h \\ -6h & 0 & 6h & -h^2 \\ -30 & -6h & 30 & 6h \\ 6h & h^2 & -6h & 0 \end{bmatrix}$$

$$+ \frac{f\ h}{420}\begin{bmatrix} 156 & 22h & 54 & -13h \\ & 4h^2 & 13h & -3h^2 \\ \text{sym} & & 156 & -22h \\ & & & 4h^2 \end{bmatrix} \qquad (A1.2)$$

$$[k]^{(e)} = \frac{1}{h^3}\begin{bmatrix} 12 & 6h & -12 & 6h \\ & 4h^2 & -6h & 2h^2 \\ \text{sym} & & 12 & -6h \\ & & & 4h^2 \end{bmatrix}$$

$$- \frac{(u^2 - \Gamma + p(1-2\nu\mu) - \gamma)}{30h}\begin{bmatrix} 36 & 3h & -36 & 3h \\ & 4h^2 & -3h & -h^2 \\ \text{sym} & & 36 & -3h \\ & & & 4h^2 \end{bmatrix}$$

Contd.....

$$- \frac{\gamma\ x_j}{30h} \begin{bmatrix} 36 & 3h & -36 & 3h \\ & 4h^2 & -3h & -h^2 \\ & & 36 & -3h \\ \text{sym} & & & 4h^2 \end{bmatrix} - \frac{\gamma}{60} \begin{bmatrix} 36 & 6h & -36 & 0 \\ & 2h^2 & -6h & -h^2 \\ & & 36 & 0 \\ \text{sym} & & & 6h^2 \end{bmatrix}$$

$$\cdots\cdots \qquad (A1.3)$$

APPENDIX-2

BOUNDARY CONDITIONS FOR STEADY FLOW

A2.1 Simple Supports :

For pipe simply supported at extreme ends, the boundary conditions will be

$$w = 0 \qquad \frac{\partial^2 w}{\partial \xi^2} = 0 \tag{A2.1}$$

For a intermediate simple support, the boundary condition will be

$$w = 0 \tag{A2.2}$$

A2.2 Fixed Supports :

For fixed ends of the pipe, the boundary conditions will be

$$w = 0 \qquad \frac{\partial w}{\partial \xi} = 0 \tag{A2.3}$$

A2.3 Elastic Supports, Displacement Springs :

For pipe supported by a displacement spring, K_d

$$\begin{aligned} \frac{\partial^3 w}{\partial \xi^3} + \alpha_d\, w = 0 \qquad \text{at} \quad \xi = 0 \\ \frac{\partial^3 w}{\partial \xi^3} - \alpha_d\, w = 0 \qquad \text{at} \quad \xi = h \end{aligned} \tag{A2.4}$$

where

$$\alpha_d = \frac{K_d\, L^3}{E\, I} \tag{A2.5}$$

A2.4 Elastic Supports, Torsional Springs :

For pipe supported by torsional spring, K_t.

$$\frac{\partial^2 w}{\partial \xi^2} - \alpha_t \frac{\partial w}{\partial \xi} = 0 \quad \text{at} \quad \xi = 0$$

$$\frac{\partial^2 w}{\partial \xi^2} + \alpha_t \frac{\partial w}{\partial \xi} = 0 \quad \text{at} \quad \xi = h \tag{A2.6}$$

where

$$\alpha_t = \frac{K_t L}{EI} \tag{A2.7}$$

APPENDIX - 3

The matrices [A], [B], [E] and [F] can be given as

$$[A] = \frac{1}{h^3}\begin{bmatrix} 12 & 6h & -12 & 6h \\ & 4h^2 & -6h & 2h^2 \\ \text{sym} & & 12 & -6h \\ & & & 4h^2 \end{bmatrix} - \frac{(A_1+\gamma x_j)}{30h}\begin{bmatrix} 36 & 3h & -36 & 3h \\ & 4h^2 & -3h & -h^2 \\ \text{sym} & & 36 & -3h \\ & & & 4h^2 \end{bmatrix}$$

$$- \frac{A_2}{420}\begin{bmatrix} 156 & 22h & 54 & -13h \\ & 4h^2 & 13h & -3h^2 \\ & & 156 & 22h \\ & \text{sym} & & 4h^2 \end{bmatrix} - \frac{\gamma}{60}\begin{bmatrix} 36 & 6h & -36 & 0 \\ & 2h^2 & -6h & -h^2 \\ & & 36 & 0 \\ & \text{sym} & & 6h^2 \end{bmatrix} \quad \ldots \quad (A3.1)$$

$$[B] = \frac{A_3\ (1 - x_j)}{30h}\begin{bmatrix} 36 & 3h & -36 & 3h \\ & 4h^2 & -3h & -h^2 \\ \text{sym} & & 36 & -3h \\ & & & 4h^2 \end{bmatrix}$$

$$- \frac{A_3}{60}\begin{bmatrix} 36 & 6h & -36 & 0 \\ & 2h^2 & -6h & -h^2 \\ \text{sym} & & 36 & 0 \\ & & & 6h^2 \end{bmatrix} - \frac{(A_3 - A_4)}{60}\begin{bmatrix} -30 & 6h & 30 & -6h \\ -6h & 0 & 6h & -h^2 \\ -30 & -6h & 30 & 6h \\ 6h & h & -6h & 0 \end{bmatrix}$$

$$- \frac{\alpha\Omega}{2h^3}\begin{bmatrix} 12 & 6h & -12 & 6h \\ & 4h^2 & -6h & 2h^2 \\ & & 12 & -6h \\ \text{sym} & & & 4h^2 \end{bmatrix} - \frac{f\ \Omega\ h}{2 \times 420}\begin{bmatrix} 156 & 22h & 54 & -13h \\ & 4h^2 & 13h & -3h^2 \\ & & 156 & 22h \\ \text{sym} & & & 4h^2 \end{bmatrix} \quad \ldots \quad (A3.2)$$

$$[E] = \frac{1}{h^3}\begin{bmatrix} 12 & 6h & -12 & 6h \\ & 4h^2 & -6h & 2h^2 \\ & & 12 & -6h \\ \text{sym} & & & 4h^2 \end{bmatrix} - \frac{(A_5+\gamma x_j)}{30h}\begin{bmatrix} 36 & 3h & -36 & 3h \\ & 4h^2 & -3h & -h^2 \\ & & 36 & -3h \\ \text{sym} & & & 4h^2 \end{bmatrix}$$

$$- \frac{A_2 h}{420}\begin{bmatrix} 156 & 22h & 54 & -13h \\ & 4h^2 & 13h & -3h^2 \\ & & 156 & 22h \\ \text{sym} & & & 4h^2 \end{bmatrix} - \frac{\gamma}{60}\begin{bmatrix} 36 & 6h & -36 & 0 \\ & 2h^2 & -6h & -h^2 \\ & & 36 & 0 \\ \text{sym} & & & 6h^2 \end{bmatrix}$$

... (A3.3)

$$[G] = \frac{A_3\ (1 - \gamma_j)}{30\ h}\begin{bmatrix} 36 & 3h & -36 & 3h \\ & 4h^2 & -3h & -h^2 \\ & & 36 & -3h \\ \text{sym} & & & 4h^2 \end{bmatrix}$$

$$- \frac{A_3}{60}\begin{bmatrix} 36 & 6h & -36 & 6 \\ & 2h^2 & -6h & -h^2 \\ & & 36 & 0 \\ \text{sym} & & & 6h^2 \end{bmatrix} - \frac{(A_3 - A_6)}{60}\begin{bmatrix} -30 & 6h & 30 & -6h \\ -6h & 0 & 6h & -h^2 \\ -30 & -6h & 30 & 6h \\ 6h & h^2 & -6h & 0 \end{bmatrix}$$

$$+ \frac{\alpha\Omega}{2h^3}\begin{bmatrix} 12 & 5h & -12 & 6h \\ & 4h^2 & -6h & 2h^2 \\ & & 12 & -6h \\ \text{sym} & & & 4h^2 \end{bmatrix} + \frac{f\Omega}{2}\ \frac{h}{420}\begin{bmatrix} 156 & 22h & 54 & -13h \\ & 4h^2 & 13h & -3h^2 \\ & & 156 & 22h \\ \text{sym} & & & 4h^2 \end{bmatrix}$$

.....(A3.4)

APPENDIX - 4

BOUNDARY CONDITIONS FOR HARMONICALLY PERTURBED FLOW

A4.1 Simple Supports :

For pipe simply supported at extreme ends, the boundary conditions will be

$$Y = 0 \qquad\qquad Z = 0$$

$$\frac{d^2Y}{d\xi^2} = 0 \qquad\qquad \frac{d^2Z}{d\xi^2} = 0 \tag{A4.1}$$

For a intermediate simple support, the boundary conditions will be

$$Y = 0 \qquad\qquad Z = 0 \tag{A4.2}$$

A4.2 Fixed Supports :

For fixed ends of pipe, the boundary conditions will be

$$Y = 0 \qquad\qquad Z = 0$$

$$\frac{dY}{d\xi} = 0 \qquad\qquad \frac{dZ}{d\xi} = 0 \tag{A4.3}$$

A4.3 Elastic Supports, Displacement Springs :

For a displacement spring K_d

$$\frac{d^3Y}{d\xi^3} + \alpha_d\, Y = 0, \quad \frac{d^3Z}{d\xi^3} + \alpha_d\, Z = 0 \quad \text{at } \xi = 0$$

$$\frac{d^3Y}{d\xi^3} - \alpha_d\, Y = 0, \quad \frac{d^3Z}{d\xi^3} - \alpha_d\, Z = 0 \quad \text{at } \xi = h \tag{A4.4}$$

where

$$\alpha_d = \frac{K_d\, L^3}{EI} \tag{A4.5}$$

A4.4 Elastic Supports, Torsional Springs :

For pipe supported by torsional spring, K_t

$$\frac{d^2Y}{d\xi^2} - \alpha_t \frac{dY}{d\xi} = 0 , \quad \frac{d^2Z}{d\xi^2} - \alpha_t \frac{dZ}{d\xi} = 0 \quad \text{at } \xi = 0$$

$$\frac{d^2Y}{d\xi^2} + \alpha_t \frac{dY}{d\xi} = 0 , \quad \frac{d^2Z}{d\xi^2} + \alpha_t \frac{dZ}{d\xi} = 0 \quad \text{at } \xi = h \qquad \ldots \qquad (A4.6)$$

where

$$\alpha_t = \frac{K_t L}{EI} \qquad (A4.7)$$